

Package ‘cml’

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Type Package

Title Conditional Manifold Learning

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Imports vegan

Description Finds a low-dimensional embedding of high-dimensional data, conditioning on available manifold information.

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Description

Find a low-dimensional embedding of high-dimensional data, conditioning on auxiliary manifold information. The current version supports conditional MDS and conditional ISOMAP.

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Details

Brief descriptions of the main functions of the package are provided below:

`condMDS()`: is the conditional MDS method, which uses conditional SMACOF to optimize its conditional stress function.

`condIsomap()`: is the conditional ISOMAP method, which is basically conditional MDS applying to graph distances (i.e., estimated geodesic distances) of the given distances/dissimilarities.

Author(s)

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References

Bui, A.T. (2024). Dimension Reduction with Prior Information for Knowledge Discovery. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 46, 3625-3636. <https://doi.org/10.1109/TPAMI.2023.3346212>

Bui, A. T. (2022). A Closed-Form Solution for Conditional Multidimensional Scaling. *Pattern Recognition Letters* 164, 148-152. <https://doi.org/10.1016/j.patrec.2022.11.007>

Bui, A.T. (2026). Conditional Multidimensional Scaling for Incomplete Conditioning Data. *Journal of Multivariate Analysis*. Accepted.

Examples

```
# Generate car-brand perception data
factor.weights <- c(90, 88, 83, 82, 81, 70, 68)/562
N <- 100
set.seed(1)
data <- matrix(runif(N*7), N, 7)
colnames(data) <- c('Quality', 'Safety', 'Value', 'Performance', 'Eco', 'Design', 'Tech')
rownames(data) <- paste('Brand', 1:N)
data.hat <- data + matrix(rnorm(N*7), N, 7)*data*.05
data.weighted <- t(apply(data, 1, function(x) x*factor.weights))
d <- dist(data.weighted)
d.hat <- d + rnorm(length(d))*d*.05
```

```

# Conditional MDS, using the first 4 factors as known features
u.cmds = condMDS(d.hat, data.hat[,1:4], 3, init='eigen')
u.cmds$B # compare with diag(factor.weights[1:4])
ccor(data.hat[,5:7], u.cmds$U)$cancor # canonical correlations
vegan::procrustes(data.hat[,5:7], u.cmds$U, symmetric = TRUE)$ss # Procrustes statistic

# Conditional ISOMAP, using the first 4 factors as known features
u.cisomap = condIsomap(d.hat, data.hat[,1:4], 3, k = 20, init='eigen')
u.cisomap$B # compare with diag(factor.weights[1:4])
ccor(data.hat[,5:7], u.cisomap$U)$cancor
vegan::procrustes(data.hat[,5:7], u.cisomap$U, symmetric = TRUE)$ss

# with missing values in V
V = data.hat[,1:4]
V[1, 1] = NA
u.cmds2 = condMDS(d.hat, V, 3, init='eigen')
u.cmds2$B # compare with diag(factor.weights[1:4])
ccor(data.hat[,5:7], u.cmds2$U)$cancor # canonical correlations
vegan::procrustes(data.hat[,5:7], u.cmds2$U, symmetric = TRUE)$ss # Procrustes statistic
u.cmds2$V.hat[1,1] # imputed value for V[1, 1]; the ground truth is data[1,1]

```

ccor

Canonical Correlations

Description

Computes canonical correlations for two sets of multivariate data x and y .

Usage

```
ccor(x, y)
```

Arguments

x	the first multivariate dataset.
y	the second multivariate dataset.

Value

a list of the following components:

cancor	a vector of canonical correlations.
xcoef	a matrix, each column of which is the vector of coefficients of x to produce the corresponding canonical covariate.
ycoef	a matrix, each column of which is the vector of coefficients of y to produce the corresponding canonical covariate.

Author(s)

Anh Tuan Bui

Examples

```
ccor(iris[,1:2], iris[,3:4])
```

condDist

Conditional Euclidean distance

Description

Internal functions.

Usage

```
condDist(U, V.tilde, one_n_t=t(rep(1,nrow(U))))  
condDist2(U, V.tilde2, one_n_t=t(rep(1,nrow(U))))
```

Arguments

U	the embedding U
V.tilde	= V %*% B
V.tilde2	= V %*% b^2*t(V)
one_n_t	= t(rep(1,nrow(U)))

Value

a dist object.

Author(s)

Anh Tuan Bui

References

Bui, A.T. (2024). Dimension Reduction with Prior Information for Knowledge Discovery. IEEE Transactions on Pattern Analysis and Machine Intelligence, 46, 3625-3636. <https://doi.org/10.1109/TPAMI.2023.3346212>

condIsomap

*Conditional ISOMAP***Description**

Finds a low-dimensional manifold embedding of a given distance/dissimilarity matrix, conditioning on auxiliary manifold parameters. The method applies conditional MDS (see [condMDS](#)) to a graph distance matrix computed for the given distances/dissimilarities, using the `isomap{vegan}` function.

Usage

```
condIsomap(d, V, u.dim, epsilon = NULL, k, W,
           method = c('matrix', 'vector'), exact = TRUE,
           it.max = 1000, gamma = 1e-05,
           init = c('none', 'condSmacof', 'eigen', 'user'),
           U.start, B.start,
           V.tilde.start = NULL, ...)
```

Arguments

<code>d</code>	a distance/dissimilarity matrix of N entities (or a <code>dist</code> object).
<code>V</code>	an $N \times q$ matrix of q manifold auxiliary parameter values of the N entities.
<code>u.dim</code>	the embedding dimension.
<code>epsilon</code>	shortest dissimilarity retained.
<code>k</code>	Number of shortest dissimilarities retained for a point. If both <code>epsilon</code> and <code>k</code> are given, <code>epsilon</code> will be used.
<code>W</code>	an $N \times N$ symmetric weight matrix. If not given, a matrix of ones will be used.
<code>method</code>	if <code>matrix</code> , there are no restrictions for the <code>B</code> matrix. If <code>vector</code> , the <code>B</code> matrix is restricted to be diagonal. The latter is more efficient for large q .
<code>exact</code>	If <code>FALSE</code> , use the large- N approximation formula to update the embedding.
<code>it.max</code>	the max number of conditional SMACOF iterations.
<code>gamma</code>	conditional SMACOF stops early if the reduction of normalized conditional stress is less than <code>gamma</code>
<code>init</code>	initialization method.
<code>U.start</code>	user-defined starting values for the embedding (when <code>init = 'user'</code>)
<code>B.start</code>	starting <code>B</code> matrix.
<code>V.tilde.start</code>	starting <code>V.tilde</code> matrix.
<code>...</code>	other arguments for the <code>isomap{vegan}</code> function.

Value

U	the embedding result.
B	the estimated B matrix.
stress	Normalized conditional stress value.
sigma	the conditional stress value at each iteration.
init	the value of the init argument.
U.start	the starting values for the embedding.
B.start	starting values for the B matrix.
V.tilde.start	starting values for the V.tilde matrix.
V.hat	imputed V matrix, if V contains rows with missing values.
method	the value of the method argument.
exact	the value of the exact argument.

Author(s)

Anh Tuan Bui

References

- Bui, A.T. (2024). Dimension Reduction with Prior Information for Knowledge Discovery. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 46, 3625-3636. <https://doi.org/10.1109/TPAMI.2023.3346212>
- Bui, A. T. (2022). A Closed-Form Solution for Conditional Multidimensional Scaling. *Pattern Recognition Letters* 164, 148-152. <https://doi.org/10.1016/j.patrec.2022.11.007>
- Bui, A.T. (2026). Conditional Multidimensional Scaling for Incomplete Conditioning Data. *Journal of Multivariate Analysis*. Accepted.

See Also

[condMDS](#)

Examples

```
# see help(cml)
```

condMDS

*Conditional Multidimensional Scaling***Description**

Wrapper of condSmacof, which finds a low-dimensional embedding of a given distance/dissimilarity matrix, conditioning on auxiliary manifold parameters.

Usage

```
condMDS(d, V, u.dim, W,
        method = c('matrix', 'vector'), exact = TRUE,
        it.max = 1000, gamma = 1e-05,
        init = c('none', 'eigen', 'user'),
        U.start, B.start, V.tilde.start = NULL)
```

Arguments

d	a distance/dissimilarity matrix of N entities (or a dist object).
V	an $N \times q$ matrix of q manifold auxiliary parameter values of the N entities.
u.dim	the embedding dimension.
W	an $N \times N$ symmetric weight matrix. If not given, a matrix of ones will be used.
method	if <i>matrix</i> , there are no restrictions for the B matrix . If <i>vector</i> , the B matrix is restricted to be diagonal. The latter is more efficient for large q .
exact	If FALSE, use the large- N approximation formula to update the embedding.
it.max	the max number of conditional SMACOF iterations.
gamma	conditional SMACOF stops early if the reduction of normalized conditional stress is less than gamma
init	initialization method.
U.start	user-defined starting values for the embedding (when <i>init</i> = 'user')
B.start	starting B matrix.
V.tilde.start	starting V.tilde matrix.

Value

U	the embedding result.
B	the estimated B matrix.
stress	Normalized conditional stress value.
sigma	the conditional stress value at each iteration.
init	the value of the <i>init</i> argument.
U.start	the starting values for the embedding.
B.start	starting values for the B matrix.

`V.tilde.start` starting values for the `V.tilde` matrix.
`V.hat` imputed `V` matrix, if `V` contains rows with missing values.
`method` the value of the `method` argument.
`exact` the value of the `exact` argument.

Author(s)

Anh Tuan Bui

References

Bui, A.T. (2024). Dimension Reduction with Prior Information for Knowledge Discovery. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 46, 3625-3636. <https://doi.org/10.1109/TPAMI.2023.3346212>

Bui, A. T. (2022). A Closed-Form Solution for Conditional Multidimensional Scaling. *Pattern Recognition Letters* 164, 148-152. <https://doi.org/10.1016/j.patrec.2022.11.007>

Bui, A.T. (2026). Conditional Multidimensional Scaling for Incomplete Conditioning Data. *Journal of Multivariate Analysis*. Accepted.

See Also

[condSmacof](#), [condSmacof_mer](#), [condMDSeigen](#), [condIsomap](#)

Examples

```
# see help(cml)
```

condMDSeigen

Conditional Multidimensional Scaling With Closed-Form Solution

Description

Provides a closed-form solution for conditional multidimensional scaling, based on multiple linear regression and eigendecomposition.

Usage

```
condMDSeigen(d, V, u.dim, method = c('matrix', 'vector'))
```

Arguments

`d` a `dist` object of N entities.
`V` an $N \times q$ matrix of q manifold auxiliary parameter values of the N entities.
`u.dim` the embedding dimension.
`method` if `matrix`, there are no restrictions for the `B` matrix . If `vector`, the `B` matrix is restricted to be diagonal.

Value

U	the embedding result.
B	the estimated B matrix.
eig	the computed eigenvalues.
stress	the corresponding normalized conditional stress value of the solution.

Author(s)

Anh Tuan Bui

References

Bui, A. T. (2022). A Closed-Form Solution for Conditional Multidimensional Scaling. *Pattern Recognition Letters* 164, 148-152. <https://doi.org/10.1016/j.patrec.2022.11.007>

See Also

[condMDS](#), [condIsomap](#)

Examples

```
# see help(cml)
```

condSmacof	<i>Conditional SMACOF</i>
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Description

The conditional SMACOF algorithm. Intended for internal usage.

Usage

```
condSmacof(d, V, u.dim, W,
           method = c('matrix', 'vector'), exact = TRUE,
           it.max = 1000, gamma = 1e-05,
           init = c('none', 'eigen', 'user'),
           U.start, B.start)
```

Arguments

d	a <i>dist</i> object of N entities.
V	an $N \times q$ matrix of q manifold auxiliary parameter values of the N entities.
u.dim	the embedding dimension.
W	an $N \times N$ symmetric weight matrix. If not given, a matrix of ones will be used.
method	if <i>matrix</i> , there are no restrictions for the B matrix . If <i>vector</i> , the B matrix is restricted to be diagonal. The latter is more efficient for large q .

<code>exact</code>	If FALSE, use the large- N approximation formula to update the embedding.
<code>it.max</code>	the max number of iterations.
<code>gamma</code>	stops early if the reduction of normalized conditional stress is less than gamma
<code>init</code>	initialization method.
<code>U.start</code>	user-defined starting values for the embedding (when <code>init = 'user'</code>).
<code>B.start</code>	user-defined starting values for the B matrix (when <code>init = 'user'</code>).

Value

<code>U</code>	the embedding result.
<code>B</code>	the estimated B matrix.
<code>stress</code>	normalized conditional stress value.
<code>sigma</code>	the conditional stress value at each iteration.
<code>init</code>	the value of the <code>init</code> argument.
<code>U.start</code>	the starting values for the embedding.
<code>B.start</code>	starting values for the B matrix.
<code>method</code>	the value of the <code>method</code> argument.

Author(s)

Anh Tuan Bui

References

- Bui, A.T. (2024). Dimension Reduction with Prior Information for Knowledge Discovery. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 46, 3625-3636. <https://doi.org/10.1109/TPAMI.2023.3346212>
- Bui, A. T. (2022). A Closed-Form Solution for Conditional Multidimensional Scaling. *Pattern Recognition Letters* 164, 148-152. <https://doi.org/10.1016/j.patrec.2022.11.007>

<code>condSmacof_mer</code>	<i>Conditional SMACOF with incomplete conditioning data (missing entire row)</i>
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Description

Conditional SMACOF with missing known feature values. If a row has a missing value in a known feature, the algorithm treats all other known feature values of that row as missing. The algorithm also imputes the actual missing value(s). Intended for internal usage.

Usage

```
condSmacof_mer(d, V, u.dim, W,
               method = c('matrix', 'vector'), exact = TRUE,
               it.max = 1000, gamma = 1e-05,
               init = c('none', 'eigen', 'user'),
               U.start, B.start, V.tilde.start)
```

Arguments

<code>d</code>	a <code>dist</code> object of N entities.
<code>V</code>	an $N \times q$ matrix of q manifold auxiliary parameter values of the N entities.
<code>u.dim</code>	the embedding dimension.
<code>W</code>	an $N \times N$ symmetric weight matrix. If not given, a matrix of ones will be used.
<code>method</code>	if <code>matrix</code> , there are no restrictions for the <code>B</code> matrix. If <code>vector</code> , the <code>B</code> matrix is restricted to be diagonal. The latter is more efficient for large q . <code>method = 'vector'</code> is not available in this version.
<code>exact</code>	If <code>FALSE</code> , use the large- N approximation formula to update the embedding.
<code>it.max</code>	the max number of iterations.
<code>gamma</code>	stops early if the reduction of normalized conditional stress is less than <code>gamma</code>
<code>init</code>	initialization method. Except for <code>init = 'user'</code> , in which case the user directly provides starting values, the other cases apply <code>condMDS</code> with <code>init = 'none'</code> (if <code>init = 'none'</code>) or <code>condMDSeigen</code> (if <code>init = 'eigen'</code>) to the complete data to initialize <code>B</code> and part of the embedding.
<code>U.start</code>	user-defined starting values for the embedding (when <code>init = 'user'</code>).
<code>B.start</code>	user-defined starting values for the <code>B</code> matrix (when <code>init = 'user'</code>).
<code>V.tilde.start</code>	user-defined starting values for the <code>V.tilde</code> matrix (when <code>init = 'user'</code>).

Value

<code>U</code>	the embedding result.
<code>B</code>	the estimated <code>B</code> matrix.
<code>stress</code>	normalized conditional stress value.
<code>sigma</code>	the conditional stress value at each iteration.
<code>init</code>	the value of the <code>init</code> argument.
<code>U.start</code>	the starting values for the embedding.
<code>B.start</code>	the starting values for the <code>B</code> matrix.
<code>V.tilde.start</code>	the starting values for <code>V.tilde.start</code> .
<code>V.hat</code>	imputed <code>V</code> matrix, if <code>V</code> contains rows with missing values.
<code>method</code>	the value of the <code>method</code> argument.

Author(s)

Anh Tuan Bui

References

- Bui, A.T. (2024). Dimension Reduction with Prior Information for Knowledge Discovery. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 46, 3625-3636. <https://doi.org/10.1109/TPAMI.2023.3346212>
- Bui, A. T. (2022). A Closed-Form Solution for Conditional Multidimensional Scaling. *Pattern Recognition Letters* 164, 148-152. <https://doi.org/10.1016/j.patrec.2022.11.007>
- Bui, A.T. (2026). Conditional Multidimensional Scaling for Incomplete Conditioning Data. *Journal of Multivariate Analysis*. Accepted.

cz	$C(Z)$
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Description

Internal function.

Usage

cz(w, d, dz)

Arguments

w	the <code>dist</code> object of a weight matrix.
d	the <code>dist</code> object of a distance/dissimilarity matrix.
dz	the <code>dist</code> object of conditional distances.

Value

the matrix $C(Z)$

Author(s)

Anh Tuan Bui

References

Bui, A.T. (2024). Dimension Reduction with Prior Information for Knowledge Discovery. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 46, 3625-3636. <https://doi.org/10.1109/TPAMI.2023.3346212>

mpinv	<i>Moore-Penrose Inverse</i>
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Description

Computes the Moore-Penrose inverse (a.k.a., generalized inverse or pseudoinverse) of a matrix based on singular-value decomposition (SVD).

Usage

mpinv(A, eps = sqrt(.Machine\$double.eps))

Arguments

A	a matrix of real numbers.
eps	a threshold (to be multiplied with the largest singular value) for dropping SVD parts that correspond to small singular values.

Value

the Moore-Penrose inverse.

Author(s)

Anh Tuan Bui

Examples

```
mpinv(2*diag(4))
```

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