GNU Linear Programming Kit

Reference Manual

Version 4.40

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Contents

Chapter 1

Introduction

GLPK (GNU Linear Programming Kit) is a set of routines written in the ANSI C programming language and organized in the form of a callable library. It is intended for solving linear programming (LP), mixed integer programming (MIP), and other related problems.

1.1 LP problem

GLPK assumes the following formulation of *linear programming (LP)* problem:

minimize (or maximize)

$$z = c_1 x_{m+1} + c_2 x_{m+2} + \ldots + c_n x_{m+n} + c_0 \tag{1.1}$$

subject to linear constraints

$$\begin{aligned} x_1 &= a_{11} x_{m+1} + a_{12} x_{m+2} + \ldots + a_{1n} x_{m+n} \\ x_2 &= a_{21} x_{m+1} + a_{22} x_{m+2} + \ldots + a_{2n} x_{m+n} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_m &= a_{m1} x_{m+1} + a_{m2} x_{m+2} + \ldots + a_{mn} x_{m+n} \end{aligned} \tag{1.2}$$

and bounds of variables

$$\begin{aligned} l_1 \leq \ & x_1 \ \leq u_1 \\ l_2 \leq \ & x_2 \ \leq u_2 \\ & \ldots\ldots\ldots \\ l_{m+n} \leq \ & x_{m+n} \leq u_{m+n} \end{aligned} \tag{1.3}$$

where: $x_1, x_2, \ldots, x_m$ are auxiliary variables; $x_{m+1}, x_{m+2}, \ldots, x_{m+n}$ are structural variables; z is the objective function; $c_1, c_2, \ldots, c_n$ are objective coefficients; c_0 is the constant term ("shift") of the objective function; $a_{11}, a_{12}, \ldots, a_{mn}$ are constraint coefficients; $l_1, l_2, \ldots, l_{m+n}$ are lower bounds of variables; $u_1, u_2, \ldots, u_{m+n}$ are upper bounds of variables.

Auxiliary variables are also called *rows*, because they correspond to rows of the constraint matrix (i.e. a matrix built of the constraint coefficients). Similarly, structural variables are also called *columns*, because they correspond to columns of the constraint matrix.

Bounds of variables can be finite as well as infinite. Besides, lower and upper bounds can be equal to each other. Thus, the following types of variables are possible:

Bounds of variable	Type of variable
$-\infty < x_k < +\infty$	Free (unbounded) variable
$l_k \leq x_k < +\infty$	Variable with lower bound
$-\infty < x_k \leq u_k$	Variable with upper bound
$l_k \leq x_k \leq u_k$	Double-bounded variable
$l_k = x_k = u_k$	Fixed variable

Note that the types of variables shown above are applicable to structural as well as to auxiliary variables.

To solve the LP problem (1.1)—(1.3) is to find such values of all structural and auxiliary variables, which:

- satisfy to all the linear constraints (1.2), and
- are within their bounds (1.3), and
- provide the smallest (in case of minimization) or the largest (in case of maximization) value of the objective function (1.1).

1.2 MIP problem

Mixed integer linear programming (MIP) problem is LP problem in which some variables are additionally required to be integer.

GLPK assumes that MIP problem has the same formulation as ordinary (pure) LP problem (1.1)—(1.3), i.e. includes auxiliary and structural variables, which may have lower and/or upper bounds. However, in case of MIP problem some variables may be required to be integer. This additional constraint means that a value of each *integer variable* must be only integer number. (Should note that GLPK allows only structural variables to be of integer kind.)

1.3 Using the package

1.3.1 Brief example

In order to understand what GLPK is from the user's standpoint, consider the following simple LP problem:

maximize

$$z = 10x_1 + 6x_2 + 4x_3$$

subject to

$$\begin{aligned} x_1 + \ \ x_2 + \ \ x_3 &\leq 100 \\ 10x_1 + 4x_2 + 5x_3 &\leq 600 \\ 2x_1 + 2x_2 + 6x_3 &\leq 300 \end{aligned}$$

where all variables are non-negative

$$x_1 \geq 0, \ x_2 \geq 0, \ x_3 \geq 0$$

At first this LP problem should be transformed to the standard form (1.1)—(1.3). This can be easily done by introducing auxiliary variables, by one for each original inequality constraint. Thus, the problem can be reformulated as follows:

maximize

$$z = 10x_1 + 6x_2 + 4x_3$$

subject to

$$\begin{aligned} p &= \ \ x_1 + \ \ x_2 + \ \ x_3 \\ q &= 10x_1 + 4x_2 + 5x_3 \\ r &= \ \ 2x_1 + 2x_2 + 6x_3 \end{aligned}$$

and bounds of variables

$$\begin{aligned} -\infty < p \leq 100 \qquad & 0 \leq x_1 < +\infty \\ -\infty < q \leq 600 \qquad & 0 \leq x_2 < +\infty \\ -\infty < r \leq 300 \qquad & 0 \leq x_3 < +\infty \end{aligned}$$

where p, q, r are auxiliary variables (rows), and x_1, x_2, x_3 are structural variables (columns).

The example C program shown below uses GLPK API routines in order to solve this LP problem.[1]

[1] If you just need to solve LP or MIP instance, you may write it in MPS or CPLEX LP format and then use the GLPK stand-alone solver to obtain a solution. This is much less time-consuming than programming in C with GLPK API routines.

```
/* sample.c */

#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

int main(void)
{     glp_prob *lp;
      int ia[1+1000], ja[1+1000];
      double ar[1+1000], z, x1, x2, x3;
s1:   lp = glp_create_prob();
s2:   glp_set_prob_name(lp, "sample");
s3:   glp_set_obj_dir(lp, GLP_MAX);
s4:   glp_add_rows(lp, 3);
s5:   glp_set_row_name(lp, 1, "p");
s6:   glp_set_row_bnds(lp, 1, GLP_UP, 0.0, 100.0);
s7:   glp_set_row_name(lp, 2, "q");
s8:   glp_set_row_bnds(lp, 2, GLP_UP, 0.0, 600.0);
s9:   glp_set_row_name(lp, 3, "r");
s10:  glp_set_row_bnds(lp, 3, GLP_UP, 0.0, 300.0);
s11:  glp_add_cols(lp, 3);
s12:  glp_set_col_name(lp, 1, "x1");
s13:  glp_set_col_bnds(lp, 1, GLP_LO, 0.0, 0.0);
s14:  glp_set_obj_coef(lp, 1, 10.0);
s15:  glp_set_col_name(lp, 2, "x2");
s16:  glp_set_col_bnds(lp, 2, GLP_LO, 0.0, 0.0);
s17:  glp_set_obj_coef(lp, 2, 6.0);
s18:  glp_set_col_name(lp, 3, "x3");
s19:  glp_set_col_bnds(lp, 3, GLP_LO, 0.0, 0.0);
s20:  glp_set_obj_coef(lp, 3, 4.0);
s21:  ia[1] = 1, ja[1] = 1, ar[1] =  1.0; /* a[1,1] =  1 */
s22:  ia[2] = 1, ja[2] = 2, ar[2] =  1.0; /* a[1,2] =  1 */
s23:  ia[3] = 1, ja[3] = 3, ar[3] =  1.0; /* a[1,3] =  1 */
s24:  ia[4] = 2, ja[4] = 1, ar[4] = 10.0; /* a[2,1] = 10 */
s25:  ia[5] = 3, ja[5] = 1, ar[5] =  2.0; /* a[3,1] =  2 */
s26:  ia[6] = 2, ja[6] = 2, ar[6] =  4.0; /* a[2,2] =  4 */
s27:  ia[7] = 3, ja[7] = 2, ar[7] =  2.0; /* a[3,2] =  2 */
s28:  ia[8] = 2, ja[8] = 3, ar[8] =  5.0; /* a[2,3] =  5 */
s29:  ia[9] = 3, ja[9] = 3, ar[9] =  6.0; /* a[3,3] =  6 */
s30:  glp_load_matrix(lp, 9, ia, ja, ar);
```

```
s31:  glp_simplex(lp, NULL);
s32:  z = glp_get_obj_val(lp);
s33:  x1 = glp_get_col_prim(lp, 1);
s34:  x2 = glp_get_col_prim(lp, 2);
s35:  x3 = glp_get_col_prim(lp, 3);
s36:  printf("\nz = %g; x1 = %g; x2 = %g; x3 = %g\n",
         z, x1, x2, x3);
s37:  glp_delete_prob(lp);
      return 0;
}

/* eof */
```

The statement `s1` creates a problem object. Being created the object is initially empty. The statement `s2` assigns a symbolic name to the problem object.

The statement `s3` calls the routine `glp_set_obj_dir` in order to set the optimization direction flag, where `GLP_MAX` means maximization.

The statement `s4` adds three rows to the problem object.

The statement `s5` assigns the symbolic name '`p`' to the first row, and the statement `s6` sets the type and bounds of the first row, where `GLP_UP` means that the row has an upper bound. The statements `s7`, `s8`, `s9`, `s10` are used in the same way in order to assign the symbolic names '`q`' and '`r`' to the second and third rows and set their types and bounds.

The statement `s11` adds three columns to the problem object.

The statement `s12` assigns the symbolic name '`x1`' to the first column, the statement `s13` sets the type and bounds of the first column, where `GLP_LO` means that the column has an lower bound, and the statement `s14` sets the objective coefficient for the first column. The statements `s15`—`s20` are used in the same way in order to assign the symbolic names '`x2`' and '`x3`' to the second and third columns and set their types, bounds, and objective coefficients.

The statements `s21`—`s29` prepare non-zero elements of the constraint matrix (i.e. constraint coefficients). Row indices of each element are stored in the array `ia`, column indices are stored in the array `ja`, and numerical values of corresponding elements are stored in the array `ar`. Then the statement `s30` calls the routine `glp_load_matrix`, which loads information from these three arrays into the problem object.

Now all data have been entered into the problem object, and therefore the statement `s31` calls the routine `glp_simplex`, which is a driver to the

simplex method, in order to solve the LP problem. This routine finds an optimal solution and stores all relevant information back into the problem object.

The statement `s32` obtains a computed value of the objective function, and the statements `s33`—`s35` obtain computed values of structural variables (columns), which correspond to the optimal basic solution found by the solver.

The statement `s36` writes the optimal solution to the standard output. The printout may look like follows:

```
*     0:   objval =   0.000000000e+00   infeas =   0.000000000e+00 (0)
*     2:   objval =   7.333333333e+02   infeas =   0.000000000e+00 (0)
OPTIMAL SOLUTION FOUND

z = 733.333; x1 = 33.3333; x2 = 66.6667; x3 = 0
```

Finally, the statement `s37` calls the routine `glp_delete_prob`, which frees all the memory allocated to the problem object.

1.3.2 Compiling

The GLPK package has the only header file `glpk.h`, which should be available on compiling a C (or C++) program using GLPK API routines.

If the header file is installed in the default location `/usr/local/include`, the following typical command may be used to compile, say, the example C program described above with the GNU C compiler:

```
$ gcc -c sample.c
```

If `glpk.h` is not in the default location, the corresponding directory containing it should be made known to the C compiler through `-I` option, for example:

```
$ gcc -I/foo/bar/glpk-4.15/include -c sample.c
```

In any case the compilation results in an object file `sample.o`.

1.3.3 Linking

The GLPK library is a single file `libglpk.a`. (On systems which support shared libraries there may be also a shared version of the library `libglpk.so`.)

If the library is installed in the default location `/usr/local/lib`, the following typical command may be used to link, say, the example C program described above against with the library:

```
$ gcc sample.o -lglpk -lm
```

If the GLPK library is not in the default location, the corresponding directory containing it should be made known to the linker through `-L` option, for example:

```
$ gcc -L/foo/bar/glpk-4.15 sample.o -lglpk -lm
```

Depending on configuration of the package linking against with the GLPK library may require the following optional libraries:

`-lgmp`	the GNU MP bignum library;
`-lz`	the zlib data compression library;
`-lltdl`	the GNU ltdl shared support library.

in which case corresponding libraries should be also made known to the linker, for example:

```
$ gcc sample.o -lglpk -lz -lltdl -lm
```

For more details about configuration options of the GLPK package see Appendix A, page 230.

Chapter 2

Basic API Routines

This chapter describes GLPK API routines intended for using in application programs.

Library header

All GLPK API data types and routines are defined in the header file `glpk.h`. It should be included in all source files which use GLPK API, either directly or indirectly through some other header file as follows:

```
#include <glpk.h>
```

Error handling

If some GLPK API routine detects erroneous or incorrect data passed by the application program, it writes appropriate diagnostic messages to the terminal and then abnormally terminates the application program. In most practical cases this allows to simplify programming by avoiding numerous checks of return codes. Thus, in order to prevent crashing the application program should check all data, which are suspected to be incorrect, before calling GLPK API routines.

Should note that this kind of error handling is used only in cases of incorrect data passed by the application program. If, for example, the application program calls some GLPK API routine to read data from an input file and these data are incorrect, the GLPK API routine reports about error in the usual way by means of the return code.

Thread safety

Currently GLPK API routines are non-reentrant and therefore cannot be used in multi-threaded programs.

Array indexing

Normally all GLPK API routines start array indexing from 1, not from 0 (except the specially stipulated cases). This means, for example, that if some vector x of the length n is passed as an array to some GLPK API routine, the latter expects vector components to be placed in locations `x[1]`, `x[2]`, ..., `x[n]`, and the location `x[0]` normally is not used.

In order to avoid indexing errors it is most convenient and most reliable to declare the array `x` as follows:

```
double x[1+n];
```

or to allocate it as follows:

```
double *x;
. . .
x = calloc(1+n, sizeof(double));
```

In both cases one extra location `x[0]` is reserved that allows passing the array to GLPK routines in a usual way.

2.1 Problem object

All GLPK API routines deal with so called *problem object*, which is a program object of type `glp_prob` and intended to represent a particular LP or MIP instance.

The type `glp_prob` is a data structure declared in the header file `glpk.h` as follows:

```
typedef struct { ... } glp_prob;
```

Problem objects (i.e. program objects of the `glp_prob` type) are allocated and managed internally by the GLPK API routines. The application program should never use any members of the `glp_prob` structure directly and should deal only with pointers to these objects (that is, `glp_prob *` values).

The problem object consists of five segments, which are:

- problem segment,
- basis segment,
- interior point segment,
- MIP segment, and
- control parameters and statistics segment.

Problem segment

The *problem segment* contains original LP/MIP data, which corresponds to the problem formulation (1.1)—(1.3) (see Section 1.1, page 13). It includes the following components:

- rows (auxiliary variables),
- columns (structural variables),
- objective function, and
- constraint matrix.

Rows and columns have the same set of the following attributes:

- ordinal number,
- symbolic name (1 up to 255 arbitrary graphic characters),
- type (free, lower bound, upper bound, double bound, fixed),
- numerical values of lower and upper bounds,
- scale factor.

Ordinal numbers are intended for referencing rows and columns. Row ordinal numbers are integers $1, 2, \ldots, m$, and column ordinal numbers are integers $1, 2, \ldots, n$, where m and n are, respectively, the current number of rows and columns in the problem object.

Symbolic names are intended for informational purposes. They also can be used for referencing rows and columns.

Types and bounds of rows (auxiliary variables) and columns (structural variables) are explained above (see Section 1.1, page 13).

Scale factors are used internally for scaling rows and columns of the constraint matrix.

Information about the *objective function* includes numerical values of objective coefficients and a flag, which defines the optimization direction (i.e. minimization or maximization).

The *constraint matrix* is a $m \times n$ rectangular matrix built of constraint coefficients a_{ij}, which defines the system of linear constraints (1.2) (see Section 1.1, page 13). This matrix is stored in the problem object in both row-wise and column-wise sparse formats.

Once the problem object has been created, the application program can access and modify any components of the problem segment in arbitrary order.

Basis segment

The *basis segment* of the problem object keeps information related to the current basic solution. It includes:

- row and column statuses,
- basic solution statuses,
- factorization of the current basis matrix, and
- basic solution components.

The *row and column statuses* define which rows and columns are basic and which are non-basic. These statuses may be assigned either by the application program of by some API routines. Note that these statuses are always defined independently on whether the corresponding basis is valid or not.

The *basic solution statuses* include the *primal status* and the *dual status*, which are set by the simplex-based solver once the problem has been solved. The primal status shows whether a primal basic solution is feasible, infeasible, or undefined. The dual status shows the same for a dual basic solution.

The *factorization of the basis matrix* is some factorized form (like LU-factorization) of the current basis matrix (defined by the current row and column statuses). The factorization is used by the simplex-based solver and kept when the solver terminates the search. This feature allows efficiently reoptimizing the problem after some modifications (for example, after changing some bounds or objective coefficients). It also allows performing the post-optimal analysis (for example, computing components of the simplex table, etc.).

The *basic solution components* include primal and dual values of all auxiliary and structural variables for the most recently obtained basic solution.

Interior point segment

The *interior point segment* is automatically allocated after the problem has been solved using the interior point solver. It contains interior point solution components, which include the solution status, and primal and dual values of all auxiliary and structural variables.

MIP segment

The *MIP segment* is used only for MIP problems. This segment includes:

- column kinds,
- MIP solution status, and
- MIP solution components.

The *column kinds* define which columns (i.e. structural variables) are integer and which are continuous.

The *MIP solution status* is set by the MIP solver and shows whether a MIP solution is integer optimal, integer feasible (non-optimal), or undefined.

The *MIP solution components* are computed by the MIP solver and include primal values of all auxiliary and structural variables for the most recently obtained MIP solution.

Note that in case of MIP problem the basis segment corresponds to the optimal solution of LP relaxation, which is also available to the application program.

Currently the search tree is not kept in the MIP segment. Therefore if the search has been terminated, it cannot be continued.

2.2 Problem creating and modifying routines

2.2.1 glp_create_prob—create problem object

Synopsis

```
glp_prob *glp_create_prob(void);
```

Description

The routine `glp_create_prob` creates a new problem object, which initially is "empty", i.e. has no rows and columns.

Returns

The routine returns a pointer to the created object, which should be used in any subsequent operations on this object.

2.2.2 glp_set_prob_name—assign (change) problem name

Synopsis

```
void glp_set_prob_name(glp_prob *lp, const char *name);
```

Description

The routine `glp_set_prob_name` assigns a given symbolic `name` (1 up to 255 characters) to the specified problem object.

If the parameter `name` is `NULL` or empty string, the routine erases an existing symbolic name of the problem object.

2.2.3 glp_set_obj_name—assign (change) objective function name

Synopsis

```
void glp_set_obj_name(glp_prob *lp, const char *name);
```

Description

The routine `glp_set_obj_name` assigns a given symbolic `name` (1 up to 255 characters) to the objective function of the specified problem object.

If the parameter `name` is `NULL` or empty string, the routine erases an existing symbolic name of the objective function.

2.2.4 glp_set_obj_dir—set (change) optimization direction flag

Synopsis

```
void glp_set_obj_dir(glp_prob *lp, int dir);
```

Description

The routine `glp_set_obj_dir` sets (changes) the optimization direction flag (i.e. "sense" of the objective function) as specified by the parameter `dir`:

`GLP_MIN` minimization;

`GLP_MAX` maximization.

(Note that by default the problem is minimization.)

2.2.5 glp_add_rows—add new rows to problem object

Synopsis

```
int glp_add_rows(glp_prob *lp, int nrs);
```

Description

The routine `glp_add_rows` adds `nrs` rows (constraints) to the specified problem object. New rows are always added to the end of the row list, so the ordinal numbers of existing rows are not changed.

Being added each new row is initially free (unbounded) and has empty list of the constraint coefficients.

Returns

The routine `glp_add_rows` returns the ordinal number of the first new row added to the problem object.

2.2.6 glp_add_cols—add new columns to problem object

Synopsis

```
int glp_add_cols(glp_prob *lp, int ncs);
```

Description

The routine `glp_add_cols` adds `ncs` columns (structural variables) to the specified problem object. New columns are always added to the end of the column list, so the ordinal numbers of existing columns are not changed.

Being added each new column is initially fixed at zero and has empty list of the constraint coefficients.

Returns

The routine `glp_add_cols` returns the ordinal number of the first new column added to the problem object.

2.2.7 glp_set_row_name—assign (change) row name

Synopsis

```
void glp_set_row_name(glp_prob *lp, int i, const char *name);
```

Description

The routine `glp_set_row_name` assigns a given symbolic `name` (1 up to 255 characters) to `i`-th row (auxiliary variable) of the specified problem object.

If the parameter `name` is `NULL` or empty string, the routine erases an existing name of i-th row.

2.2.8 glp_set_col_name—assign (change) column name

Synopsis

```
void glp_set_col_name(glp_prob *lp, int j, const char *name);
```

Description

The routine `glp_set_col_name` assigns a given symbolic `name` (1 up to 255 characters) to `j`-th column (structural variable) of the specified problem object.

If the parameter `name` is `NULL` or empty string, the routine erases an existing name of j-th column.

2.2.9 glp_set_row_bnds—set (change) row bounds

Synopsis

```
void glp_set_row_bnds(glp_prob *lp, int i, int type,
      double lb, double ub);
```

Description

The routine `glp_set_row_bnds` sets (changes) the type and bounds of `i`-th row (auxiliary variable) of the specified problem object.

The parameters `type`, `lb`, and `ub` specify the type, lower bound, and upper bound, respectively, as follows:

Type	Bounds	Comment
`GLP_FR`	$-\infty < x < +\infty$	Free (unbounded) variable
`GLP_LO`	$lb \leq x < +\infty$	Variable with lower bound
`GLP_UP`	$-\infty < x \leq ub$	Variable with upper bound
`GLP_DB`	$lb \leq x \leq ub$	Double-bounded variable
`GLP_FX`	$lb = x = ub$	Fixed variable

where x is the auxiliary variable associated with i-th row.

If the row has no lower bound, the parameter `lb` is ignored. If the row has no upper bound, the parameter `ub` is ignored. If the row is an equality constraint (i.e. the corresponding auxiliary variable is of fixed type), only the parameter `lb` is used while the parameter `ub` is ignored.

Being added to the problem object each row is initially free, i.e. its type is `GLP_FR`.

2.2.10 glp_set_col_bnds—set (change) column bounds

Synopsis

```
void glp_set_col_bnds(glp_prob *lp, int j, int type,
      double lb, double ub);
```

Description

The routine `glp_set_col_bnds` sets (changes) the type and bounds of `j`-th column (structural variable) of the specified problem object.

The parameters `type`, `lb`, and `ub` specify the type, lower bound, and upper bound, respectively, as follows:

Type	Bounds	Comment
`GLP_FR`	$-\infty < x < +\infty$	Free (unbounded) variable
`GLP_LO`	$lb \leq x < +\infty$	Variable with lower bound
`GLP_UP`	$-\infty < x \leq ub$	Variable with upper bound
`GLP_DB`	$lb \leq x \leq ub$	Double-bounded variable
`GLP_FX`	$lb = x = ub$	Fixed variable

where x is the structural variable associated with j-th column.

If the column has no lower bound, the parameter `lb` is ignored. If the column has no upper bound, the parameter `ub` is ignored. If the column is of fixed type, only the parameter `lb` is used while the parameter `ub` is ignored.

Being added to the problem object each column is initially fixed at zero, i.e. its type is `GLP_FX` and both bounds are 0.

2.2.11 glp_set_obj_coef—set (change) objective coefficient or constant term

Synopsis

```
void glp_set_obj_coef(glp_prob *lp, int j, double coef);
```

Description

The routine `glp_set_obj_coef` sets (changes) the objective coefficient at `j`-th column (structural variable). A new value of the objective coefficient is specified by the parameter `coef`.

If the parameter `j` is 0, the routine sets (changes) the constant term ("shift") of the objective function.

2.2.12 glp_set_mat_row—set (replace) row of the constraint matrix

Synopsis

```
void glp_set_mat_row(glp_prob *lp, int i, int len,
      const int ind[], const double val[]);
```

Description

The routine `glp_set_mat_row` stores (replaces) the contents of `i`-th row of the constraint matrix of the specified problem object.

Column indices and numerical values of new row elements must be placed in locations `ind[1]`, ..., `ind[len]` and `val[1]`, ..., `val[len]`, respectively, where $0 \leq$ `len` $\leq n$ is the new length of i-th row, n is the current number of columns in the problem object. Elements with identical column indices are not allowed. Zero elements are allowed, but they are not stored in the constraint matrix.

If the parameter `len` is 0, the parameters `ind` and/or `val` can be specified as `NULL`.

2.2.13 glp_set_mat_col—set (replace) column of the constraint matrix

Synopsis

```
void glp_set_mat_col(glp_prob *lp, int j, int len,
      const int ind[], const double val[]);
```

Description

The routine `glp_set_mat_col` stores (replaces) the contents of `j`-th column of the constraint matrix of the specified problem object.

Row indices and numerical values of new column elements must be placed in locations `ind[1]`, ..., `ind[len]` and `val[1]`, ..., `val[len]`, respectively, where $0 \leq$ `len` $\leq m$ is the new length of j-th column, m is the current number of rows in the problem object. Elements with identical row indices are not allowed. Zero elements are allowed, but they are not stored in the constraint matrix.

If the parameter `len` is 0, the parameters `ind` and/or `val` can be specified as `NULL`.

2.2.14 glp_load_matrix—load (replace) the whole constraint matrix

Synopsis

```
void glp_load_matrix(glp_prob *lp, int ne, const int ia[],
      const int ja[], const double ar[]);
```

Description

The routine `glp_load_matrix` loads the constraint matrix passed in the arrays `ia`, `ja`, and `ar` into the specified problem object. Before loading the current contents of the constraint matrix is destroyed.

Constraint coefficients (elements of the constraint matrix) must be specified as triplets (`ia[k]`, `ja[k]`, `ar[k]`) for $k = 1, \ldots, ne$, where `ia[k]` is the row index, `ja[k]` is the column index, and `ar[k]` is a numeric value of corresponding constraint coefficient. The parameter `ne` specifies the total number of (non-zero) elements in the matrix to be loaded. Coefficients with identical indices are not allowed. Zero coefficients are allowed, however, they are not stored in the constraint matrix.

If the parameter `ne` is 0, the parameters `ia`, `ja`, and/or `ar` can be specified as `NULL`.

2.2.15 glp_del_rows—delete rows from problem object

Synopsis

```
void glp_del_rows(glp_prob *lp, int nrs, const int num[]);
```

Description

The routine `glp_del_rows` deletes rows from the specified problem object. Ordinal numbers of rows to be deleted should be placed in locations `num[1]`, ..., `num[nrs]`, where `nrs` > 0.

Note that deleting rows involves changing ordinal numbers of other rows remaining in the problem object. New ordinal numbers of the remaining rows are assigned under the assumption that the original order of rows is not changed. Let, for example, before deletion there be five rows a, b, c, d, e with ordinal numbers 1, 2, 3, 4, 5, and let rows b and d have been deleted. Then after deletion the remaining rows a, c, e are assigned new oridinal numbers 1, 2, 3.

2.2.16 glp_del_cols—delete columns from problem object

Synopsis

```
void glp_del_cols(glp_prob *lp, int ncs, const int num[]);
```

Description

The routine `glp_del_cols` deletes columns from the specified problem object. Ordinal numbers of columns to be deleted should be placed in locations `num[1]`, ..., `num[ncs]`, where `ncs` > 0.

Note that deleting columns involves changing ordinal numbers of other columns remaining in the problem object. New ordinal numbers of the remaining columns are assigned under the assumption that the original order of columns is not changed. Let, for example, before deletion there be six columns p, q, r, s, t, u with ordinal numbers 1, 2, 3, 4, 5, 6, and let columns p, q, s have been deleted. Then after deletion the remaining columns r, t, u are assigned new ordinal numbers 1, 2, 3.

2.2.17 glp_copy_prob—copy problem object content

Synopsis

```
void glp_copy_prob(glp_prob *dest, glp_prob *prob, int names);
```

Description

The routine `glp_copy_prob` copies the content of the problem object `prob` to the problem object `dest`.

The parameter `names` is a flag. If it is `GLP_ON`, the routine also copies all symbolic names; otherwise, if it is `GLP_OFF`, no symbolic names are copied.

2.2.18 glp_erase_prob—erase problem object content

Synopsis

```
void glp_erase_prob(glp_prob *lp);
```

Description

The routine `glp_erase_prob` erases the content of the specified problem object. The effect of this operation is the same as if the problem object would be deleted with the routine `glp_delete_prob` and then created anew

with the routine `glp_create_prob`, with the only exception that the handle (pointer) to the problem object remains valid.

2.2.19 glp_delete_prob—delete problem object

Synopsis

```
void glp_delete_prob(glp_prob *lp);
```

Description

The routine `glp_delete_prob` deletes a problem object, which the parameter `lp` points to, freeing all the memory allocated to this object.

2.3 Problem retrieving routines

2.3.1 glp_get_prob_name—retrieve problem name

Synopsis

```
const char *glp_get_prob_name(glp_prob *lp);
```

Returns

The routine `glp_get_prob_name` returns a pointer to an internal buffer, which contains symbolic name of the problem. However, if the problem has no assigned name, the routine returns `NULL`.

2.3.2 glp_get_obj_name—retrieve objective function name

Synopsis

```
const char *glp_get_obj_name(glp_prob *lp);
```

Returns

The routine `glp_get_obj_name` returns a pointer to an internal buffer, which contains symbolic name assigned to the objective function. However, if the objective function has no assigned name, the routine returns `NULL`.

2.3.3 glp_get_obj_dir—retrieve optimization direction flag

Synopsis

```
int glp_get_obj_dir(glp_prob *lp);
```

Returns

The routine `glp_get_obj_dir` returns the optimization direction flag (i.e. "sense" of the objective function):

`GLP_MIN` minimization;
`GLP_MAX` maximization.

2.3.4 glp_get_num_rows—retrieve number of rows

Synopsis

```
int glp_get_num_rows(glp_prob *lp);
```

Returns

The routine `glp_get_num_rows` returns the current number of rows in the specified problem object.

2.3.5 glp_get_num_cols—retrieve number of columns

Synopsis

```
int glp_get_num_cols(glp_prob *lp);
```

Returns

The routine `glp_get_num_cols` returns the current number of columns the specified problem object.

2.3.6 glp_get_row_name—retrieve row name

Synopsis

```
const char *glp_get_row_name(glp_prob *lp, int i);
```

Returns

The routine `glp_get_row_name` returns a pointer to an internal buffer, which contains a symbolic name assigned to `i`-th row. However, if the row has no assigned name, the routine returns `NULL`.

2.3.7 glp_get_col_name—retrieve column name

Synopsis

```
const char *glp_get_col_name(glp_prob *lp, int j);
```

Returns

The routine `glp_get_col_name` returns a pointer to an internal buffer, which contains a symbolic name assigned to `j`-th column. However, if the column has no assigned name, the routine returns `NULL`.

2.3.8 glp_get_row_type—retrieve row type

Synopsis

```
int glp_get_row_type(glp_prob *lp, int i);
```

Returns

The routine `glp_get_row_type` returns the type of `i`-th row, i.e. the type of corresponding auxiliary variable, as follows:

`GLP_FR`	free (unbounded) variable;
`GLP_LO`	variable with lower bound;
`GLP_UP`	variable with upper bound;
`GLP_DB`	double-bounded variable;
`GLP_FX`	fixed variable.

2.3.9 glp_get_row_lb—retrieve row lower bound

Synopsis

```
double glp_get_row_lb(glp_prob *lp, int i);
```

Returns

The routine `glp_get_row_lb` returns the lower bound of `i`-th row, i.e. the lower bound of corresponding auxiliary variable. However, if the row has no lower bound, the routine returns `-DBL_MAX`.

2.3.10 glp_get_row_ub—retrieve row upper bound

Synopsis

```
double glp_get_row_ub(glp_prob *lp, int i);
```

Returns

The routine `glp_get_row_ub` returns the upper bound of `i`-th row, i.e. the upper bound of corresponding auxiliary variable. However, if the row has no upper bound, the routine returns `+DBL_MAX`.

2.3.11 glp_get_col_type—retrieve column type

Synopsis

```
int glp_get_col_type(glp_prob *lp, int j);
```

Returns

The routine `glp_get_col_type` returns the type of `j`-th column, i.e. the type of corresponding structural variable, as follows:

`GLP_FR`	free (unbounded) variable;
`GLP_LO`	variable with lower bound;
`GLP_UP`	variable with upper bound;
`GLP_DB`	double-bounded variable;
`GLP_FX`	fixed variable.

2.3.12 glp_get_col_lb—retrieve column lower bound

Synopsis

```
double glp_get_col_lb(glp_prob *lp, int j);
```

Returns

The routine `glp_get_col_lb` returns the lower bound of `j`-th column, i.e. the lower bound of corresponding structural variable. However, if the column has no lower bound, the routine returns `-DBL_MAX`.

2.3.13 glp_get_col_ub—retrieve column upper bound

Synopsis

```
double glp_get_col_ub(glp_prob *lp, int j);
```

Returns

The routine `glp_get_col_ub` returns the upper bound of `j`-th column, i.e. the upper bound of corresponding structural variable. However, if the column has no upper bound, the routine returns `+DBL_MAX`.

2.3.14 glp_get_obj_coef—retrieve objective coefficient or constant term

Synopsis

```
double glp_get_obj_coef(glp_prob *lp, int j);
```

Returns

The routine `glp_get_obj_coef` returns the objective coefficient at `j`-th structural variable (column).

If the parameter `j` is 0, the routine returns the constant term ("shift") of the objective function.

2.3.15 glp_get_num_nz—retrieve number of constraint coefficients

Synopsis

```
int glp_get_num_nz(glp_prob *lp);
```

Returns

The routine `glp_get_num_nz` returns the number of non-zero elements in the constraint matrix of the specified problem object.

2.3.16 glp_get_mat_row—retrieve row of the constraint matrix

Synopsis

```
int glp_get_mat_row(glp_prob *lp, int i, int ind[],
      double val[]);
```

Description

The routine `glp_get_mat_row` scans (non-zero) elements of `i`-th row of the constraint matrix of the specified problem object and stores their column indices and numeric values to locations `ind[1]`, ..., `ind[len]` and `val[1]`, ..., `val[len]`, respectively, where $0 \leq$ `len` $\leq n$ is the number of elements in i-th row, n is the number of columns.

The parameter `ind` and/or `val` can be specified as `NULL`, in which case corresponding information is not stored.

Returns

The routine `glp_get_mat_row` returns the length `len`, i.e. the number of (non-zero) elements in `i`-th row.

2.3.17 glp_get_mat_col—retrieve column of the constraint matrix

Synopsis

```
int glp_get_mat_col(glp_prob *lp, int j, int ind[],
      double val[]);
```

Description

The routine `glp_get_mat_col` scans (non-zero) elements of `j`-th column of the constraint matrix of the specified problem object and stores their row indices and numeric values to locations `ind[1]`, ..., `ind[len]` and `val[1]`, ..., `val[len]`, respectively, where $0 \leq$ `len` $\leq m$ is the number of elements in j-th column, m is the number of rows.

The parameter `ind` and/or `val` can be specified as `NULL`, in which case corresponding information is not stored.

Returns

The routine `glp_get_mat_col` returns the length `len`, i.e. the number of (non-zero) elements in `j`-th column.

2.4 Row and column searching routines

2.4.1 glp_create_index—create the name index

Synopsis

```
void glp_create_index(glp_prob *lp);
```

Description

The routine `glp_create_index` creates the name index for the specified problem object. The name index is an auxiliary data structure, which is intended to quickly (i.e. for logarithmic time) find rows and columns by their names.

This routine can be called at any time. If the name index already exists, the routine does nothing.

2.4.2 glp_find_row—find row by its name

Synopsis

```
int glp_find_row(glp_prob *lp, const char *name);
```

Returns

The routine `glp_find_row` returns the ordinal number of a row, which is assigned (by the routine `glp_set_row_name`) the specified symbolic `name`. If no such row exists, the routine returns 0.

2.4.3 glp_find_col—find column by its name

Synopsis

```
int glp_find_col(glp_prob *lp, const char *name);
```

Returns

The routine `glp_find_col` returns the ordinal number of a column, which is assigned (by the routine `glp_set_col_name`) the specified symbolic `name`. If no such column exists, the routine returns 0.

2.4.4 glp_delete_index—delete the name index

Synopsis

```
void glp_delete_index(glp_prob *lp);
```

Description

The routine `glp_delete_index` deletes the name index previously created by the routine `glp_create_index` and frees the memory allocated to this auxiliary data structure.

This routine can be called at any time. If the name index does not exist, the routine does nothing.

2.5 Problem scaling routines

2.5.1 Background

In GLPK the *scaling* means a linear transformation applied to the constraint matrix to improve its numerical properties.[1]

The main equality is the following:

$$\widetilde{A} = RAS, \tag{2.1}$$

where $A = (a_{ij})$ is the original constraint matrix, $R = (r_{ii}) > 0$ is a diagonal matrix used to scale rows (constraints), $S = (s_{jj}) > 0$ is a diagonal matrix used to scale columns (variables), $\widetilde{A}$ is the scaled constraint matrix.

From (2.1) it follows that in the *scaled* problem instance each original constraint coefficient a_{ij} is replaced by corresponding scaled constraint coefficient:

$$\widetilde{a}_{ij} = r_{ii} a_{ij} s_{jj}. \tag{2.2}$$

Note that the scaling is performed internally and therefore transparently to the user. This means that on API level the user always deal with unscaled data.

Scale factors r_{ii} and s_{jj} can be set or changed at any time either directly by the application program in a problem specific way (with the routines `glp_set_rii` and `glp_set_sjj`), or by some API routines intended for automatic scaling.

2.5.2 glp_set_rii—set (change) row scale factor

Synopsis

```
void glp_set_rii(glp_prob *lp, int i, double rii);
```

Description

The routine `glp_set_rii` sets (changes) the scale factor r_{ii} for i-th row of the specified problem object.

[1] In many cases a proper scaling allows making the constraint matrix to be better conditioned, i.e. decreasing its condition number, that makes computations numerically more stable.

2.5.3 glp_set_sjj—set (change) column scale factor

Synopsis

```
void glp_set_sjj(glp_prob *lp, int j, double sjj);
```

Description

The routine `glp_set_sjj` sets (changes) the scale factor s_{jj} for j-th column of the specified problem object.

2.5.4 glp_get_rii—retrieve row scale factor

Synopsis

```
double glp_get_rii(glp_prob *lp, int i);
```

Returns

The routine `glp_get_rii` returns current scale factor r_{ii} for i-th row of the specified problem object.

2.5.5 glp_get_sjj—retrieve column scale factor

Synopsis

```
double glp_get_sjj(glp_prob *lp, int j);
```

Returns

The routine `glp_get_sjj` returns current scale factor s_{jj} for j-th column of the specified problem object.

2.5.6 glp_scale_prob—scale problem data

Synopsis

```
void glp_scale_prob(glp_prob *lp, int flags);
```

Description

The routine `glp_scale_prob` performs automatic scaling of problem data for the specified problem object.

The parameter `flags` specifies scaling options used by the routine. The options can be combined with the bitwise OR operator and may be the following:

`GLP_SF_GM`	perform geometric mean scaling;
`GLP_SF_EQ`	perform equilibration scaling;
`GLP_SF_2N`	round scale factors to nearest power of two;
`GLP_SF_SKIP`	skip scaling, if the problem is well scaled.

The parameter `flags` may be specified as `GLP_SF_AUTO`, in which case the routine chooses the scaling options automatically.

2.5.7 glp_unscale_prob—unscale problem data

Synopsis

```
void glp_unscale_prob(glp_prob *lp);
```

The routine `glp_unscale_prob` performs unscaling of problem data for the specified problem object.

"Unscaling" means replacing the current scaling matrices R and S by unity matrices that cancels the scaling effect.

2.6 LP basis constructing routines

2.6.1 Background

To start the search the simplex method needs a valid initial basis. In GLPK the basis is completely defined by a set of *statuses* assigned to *all* (auxiliary and structural) variables, where the status may be one of the following:

`GLP_BS` basic variable;
`GLP_NL` non-basic variable having active lower bound;
`GLP_NU` non-basic variable having active upper bound;
`GLP_NF` non-basic free variable;
`GLP_NS` non-basic fixed variable.

The basis is *valid*, if the basis matrix, which is a matrix built of columns of the augmented constraint matrix $(I|-A)$ corresponding to basic variables, is non-singular. This, in particular, means that the number of basic variables must be the same as the number of rows in the problem object. (For more details see Section 4.2, page 108.)

Any initial basis may be constructed (or restored) with the API routines `glp_set_row_stat` and `glp_set_col_stat` by assigning appropriate statuses to auxiliary and structural variables. Another way to construct an initial basis is to use API routines like `glp_adv_basis`, which implement so called *crashing*.[2] Note that on normal exit the simplex solver remains the basis valid, so in case of reoptimization there is no need to construct an initial basis from scratch.

2.6.2 glp_set_row_stat—set (change) row status

Synopsis

```
void glp_set_row_stat(glp_prob *lp, int i, int stat);
```

Description

The routine `glp_set_row_stat` sets (changes) the current status of `i`-th row (auxiliary variable) as specified by the parameter `stat`:

`GLP_BS` make the row basic (make the constraint inactive);
`GLP_NL` make the row non-basic (make the constraint active);

[2] This term is from early linear programming systems and means a heuristic to construct a valid initial basis.

`GLP_NU` make the row non-basic and set it to the upper bound; if the row is not double-bounded, this status is equivalent to `GLP_NL` (only in the case of this routine);

`GLP_NF` the same as `GLP_NL` (only in the case of this routine);

`GLP_NS` the same as `GLP_NL` (only in the case of this routine).

2.6.3 glp_set_col_stat—set (change) column status

Synopsis

```
void glp_set_col_stat(glp_prob *lp, int j, int stat);
```

Description

The routine `glp_set_col_stat sets` (changes) the current status of `j`-th column (structural variable) as specified by the parameter `stat`:

`GLP_BS` make the column basic;

`GLP_NL` make the column non-basic;

`GLP_NU` make the column non-basic and set it to the upper bound; if the column is not double-bounded, this status is equivalent to `GLP_NL` (only in the case of this routine);

`GLP_NF` the same as `GLP_NL` (only in the case of this routine);

`GLP_NS` the same as `GLP_NL` (only in the case of this routine).

2.6.4 glp_std_basis—construct standard initial LP basis

Synopsis

```
void glp_std_basis(glp_prob *lp);
```

Description

The routine `glp_std_basis` constructs the "standard" (trivial) initial LP basis for the specified problem object.

In the "standard" LP basis all auxiliary variables (rows) are basic, and all structural variables (columns) are non-basic (so the corresponding basis matrix is unity).

2.6.5 glp_adv_basis—construct advanced initial LP basis

Synopsis

```
void glp_adv_basis(glp_prob *lp, int flags);
```

Description

The routine `glp_adv_basis` constructs an advanced initial LP basis for the specified problem object.

The parameter `flags` is reserved for use in the future and must be specified as zero.

In order to construct the advanced initial LP basis the routine does the following:

1) includes in the basis all non-fixed auxiliary variables;

2) includes in the basis as many non-fixed structural variables as possible keeping the triangular form of the basis matrix;

3) includes in the basis appropriate (fixed) auxiliary variables to complete the basis.

As a result the initial LP basis has as few fixed variables as possible and the corresponding basis matrix is triangular.

2.6.6 glp_cpx_basis—construct Bixby's initial LP basis

Synopsis

```
void glp_cpx_basis(glp_prob *lp);
```

Description

The routine `glp_cpx_basis` constructs an initial basis for the specified problem object with the algorithm proposed by R. Bixby.[3]

[3] Robert E. Bixby, "Implementing the Simplex Method: The Initial Basis." ORSA Journal on Computing, Vol. 4, No. 3, 1992, pp. 267-84.

2.7 Simplex method routines

The *simplex method* is a well known efficient numerical procedure to solve LP problems.

On each iteration the simplex method transforms the original system of equaility constraints (1.2) resolving them through different sets of variables to an equivalent system called *the simplex table* (or sometimes *the simplex tableau*), which has the following form:

$$\begin{aligned} z = {} & d_1(x_N)_1 + d_2(x_N)_2 + \ldots + d_n(x_N)_n \\ (x_B)_1 = {} & \xi_{11}(x_N)_1 + \xi_{12}(x_N)_2 + \ldots + \xi_{1n}(x_N)_n \\ (x_B)_2 = {} & \xi_{21}(x_N)_1 + \xi_{22}(x_N)_2 + \ldots + \xi_{2n}(x_N)_n \\ & \ldots\ldots\ldots\ldots\ldots \\ (x_B)_m = {} & \xi_{m1}(x_N)_1 + \xi_{m2}(x_N)_2 + \ldots + \xi_{mn}(x_N)_n \end{aligned} \tag{2.3}$$

where: $(x_B)_1, (x_B)_2, \ldots, (x_B)_m$ are basic variables; $(x_N)_1, (x_N)_2, \ldots, (x_N)_n$ are non-basic variables; $d_1, d_2, \ldots, d_n$ are reduced costs; $\xi_{11}, \xi_{12}, \ldots, \xi_{mn}$ are coefficients of the simplex table. (May note that the original LP problem (1.1)—(1.3) also has the form of a simplex table, where all equalities are resolved through auxiliary variables.)

From the linear programming theory it is known that if an optimal solution of the LP problem (1.1)—(1.3) exists, it can always be written in the form (2.3), where non-basic variables are set on their bounds while values of the objective function and basic variables are determined by the corresponding equalities of the simplex table.

A set of values of all basic and non-basic variables determined by the simplex table is called *basic solution*. If all basic variables are within their bounds, the basic solution is called *(primal) feasible*, otherwise it is called *(primal) infeasible*. A feasible basic solution, which provides a smallest (in case of minimization) or a largest (in case of maximization) value of the objective function is called *optimal*. Therefore, for solving LP problem the simplex method tries to find its optimal basic solution.

Primal feasibility of some basic solution may be stated by simple checking if all basic variables are within their bounds. Basic solution is optimal if additionally the following optimality conditions are satisfied for all non-basic variables:

Status of $(x_N)_j$	Minimization	Maximization
$(x_N)_j$ is free	$d_j = 0$	$d_j = 0$
$(x_N)_j$ is on its lower bound	$d_j \geq 0$	$d_j \leq 0$
$(x_N)_j$ is on its upper bound	$d_j \leq 0$	$d_j \geq 0$

In other words, basic solution is optimal if there is no non-basic variable, which changing in the feasible direction (i.e. increasing if it is free or on its lower bound, or decreasing if it is free or on its upper bound) can improve (i.e. decrease in case of minimization or increase in case of maximization) the objective function.

If all non-basic variables satisfy to the optimality conditions shown above (independently on whether basic variables are within their bounds or not), the basic solution is called *dual feasible*, otherwise it is called *dual infeasible*.

It may happen that some LP problem has no primal feasible solution due to incorrect formulation—this means that its constraints conflict with each other. It also may happen that some LP problem has unbounded solution again due to incorrect formulation—this means that some non-basic variable can improve the objective function, i.e. the optimality conditions are violated, and at the same time this variable can infinitely change in the feasible direction meeting no resistance from basic variables. (May note that in the latter case the LP problem has no dual feasible solution.)

2.7.1 glp_simplex—solve LP problem with the primal or dual simplex method

Synopsis

```
int glp_simplex(glp_prob *lp, const glp_smcp *parm);
```

Description

The routine `glp_simplex` is a driver to the LP solver based on the simplex method. This routine retrieves problem data from the specified problem object, calls the solver to solve the problem instance, and stores results of computations back into the problem object.

The simplex solver has a set of control parameters. Values of the control parameters can be passed in the structure `glp_smcp`, which the parameter `parm` points to. For detailed description of this structure see paragraph "Control parameters" below. Before specifying some control parameters the application program should initialize the structure `glp_smcp` by default values of all control parameters using the routine `glp_init_smcp` (see the next subsection). This is needed for backward compatibility, because in the future there may appear new members in the structure `glp_smcp`.

The parameter `parm` can be specified as `NULL`, in which case the solver uses default settings.

Returns

0	The LP problem instance has been successfully solved. (This code does *not* necessarily mean that the solver has found optimal solution. It only means that the solution process was successful.)
`GLP_EBADB`	Unable to start the search, because the initial basis specified in the problem object is invalid—the number of basic (auxiliary and structural) variables is not the same as the number of rows in the problem object.
`GLP_ESING`	Unable to start the search, because the basis matrix corresponding to the initial basis is singular within the working precision.
`GLP_ECOND`	Unable to start the search, because the basis matrix corresponding to the initial basis is ill-conditioned, i.e. its condition number is too large.
`GLP_EBOUND`	Unable to start the search, because some double-bounded (auxiliary or structural) variables have incorrect bounds.
`GLP_EFAIL`	The search was prematurely terminated due to the solver failure.
`GLP_EOBJLL`	The search was prematurely terminated, because the objective function being maximized has reached its lower limit and continues decreasing (the dual simplex only).
`GLP_EOBJUL`	The search was prematurely terminated, because the objective function being minimized has reached its upper limit and continues increasing (the dual simplex only).
`GLP_EITLIM`	The search was prematurely terminated, because the simplex iteration limit has been exceeded.
`GLP_ETMLIM`	The search was prematurely terminated, because the time limit has been exceeded.
`GLP_ENOPFS`	The LP problem instance has no primal feasible solution (only if the LP presolver is used).
`GLP_ENODFS`	The LP problem instance has no dual feasible solution (only if the LP presolver is used).

Built-in LP presolver

The simplex solver has *built-in LP presolver*. It is a subprogram that transforms the original LP problem specified in the problem object to an equivalent LP problem, which may be easier for solving with the simplex method than the original one. This is attained mainly due to reducing the prob-

lem size and improving its numeric properties (for example, by removing some inactive constraints or by fixing some non-basic variables). Once the transformed LP problem has been solved, the presolver transforms its basic solution back to the corresponding basic solution of the original problem.

Presolving is an optional feature of the routine `glp_simplex`, and by default it is disabled. In order to enable the LP presolver the control parameter `presolve` should be set to `GLP_ON` (see paragraph "Control parameters" below). Presolving may be used when the problem instance is solved for the first time. However, on performing re-optimization the presolver should be disabled.

The presolving procedure is transparent to the API user in the sense that all necessary processing is performed internally, and a basic solution of the original problem recovered by the presolver is the same as if it were computed directly, i.e. without presolving.

Note that the presolver is able to recover only optimal solutions. If a computed solution is infeasible or non-optimal, the corresponding solution of the original problem cannot be recovered and therefore remains undefined. If you need to know a basic solution even if it is infeasible or non-optimal, the presolver should be disabled.

Terminal output

Solving large problem instances may take a long time, so the solver reports some information about the current basic solution, which is sent to the terminal. This information has the following format:

```
nnn:  obj = xxx  infeas = yyy (ddd)
```

where: '`nnn`' is the iteration number, '`xxx`' is the current value of the objective function (it is is unscaled and has correct sign); '`yyy`' is the current sum of primal or dual infeasibilities (it is scaled and therefore may be used only for visual estimating), '`ddd`' is the current number of fixed basic variables.

The symbol preceding the iteration number indicates which phase of the simplex method is in effect:

Blank means that the solver is searching for primal feasible solution using the primal simplex or for dual feasible solution using the dual simplex;

Asterisk (`*`) means that the solver is searching for optimal solution using the primal simplex;

Vertical dash (`|`) means that the solver is searching for optimal solution using the dual simplex.

Control parameters

This paragraph describes all control parameters currently used in the simplex solver. Symbolic names of control parameters are names of corresponding members in the structure `glp_smcp`.

`int msg_lev` (default: `GLP_MSG_ALL`)
Message level for terminal output:
`GLP_MSG_OFF`—no output;
`GLP_MSG_ERR`—error and warning messages only;
`GLP_MSG_ON` —normal output;
`GLP_MSG_ALL`—full output (including informational messages).

`int meth` (default: `GLP_PRIMAL`)
Simplex method option:
`GLP_PRIMAL`—use two-phase primal simplex;
`GLP_DUAL` —use two-phase dual simplex;
`GLP_DUALP` —use two-phase dual simplex, and if it fails, switch to the primal simplex.

`int pricing` (default: `GLP_PT_PSE`)
Pricing technique:
`GLP_PT_STD`—standard (textbook);
`GLP_PT_PSE`—projected steepest edge.

`int r_test` (default: `GLP_RT_HAR`)
Ratio test technique:
`GLP_RT_STD`—standard (textbook);
`GLP_RT_HAR`—Harris' two-pass ratio test.

`double tol_bnd` (default: `1e-7`)
Tolerance used to check if the basic solution is primal feasible. (Do not change this parameter without detailed understanding its purpose.)

`double tol_dj` (default: `1e-7`)
Tolerance used to check if the basic solution is dual feasible. (Do not change this parameter without detailed understanding its purpose.)

`double tol_piv` (default: `1e-10`)
Tolerance used to choose eligble pivotal elements of the simplex table. (Do not change this parameter without detailed understanding its purpose.)

`double obj_ll` (default: `-DBL_MAX`)
: Lower limit of the objective function. If the objective function reaches this limit and continues decreasing, the solver terminates the search. (Used in the dual simplex only.)

`double obj_ul` (default: `+DBL_MAX`)
: Upper limit of the objective function. If the objective function reaches this limit and continues increasing, the solver terminates the search. (Used in the dual simplex only.)

`int it_lim` (default: `INT_MAX`)
: Simplex iteration limit.

`int tm_lim` (default: `INT_MAX`)
: Searching time limit, in milliseconds.

`int out_frq` (default: `200`)
: Output frequency, in iterations. This parameter specifies how frequently the solver sends information about the solution process to the terminal.

`int out_dly` (default: `0`)
: Output delay, in milliseconds. This parameter specifies how long the solver should delay sending information about the solution process to the terminal.

`int presolve` (default: `GLP_OFF`)
: LP presolver option:
`GLP_ON` —enable using the LP presolver;
`GLP_OFF`—disable using the LP presolver.

Example 1

The following main program reads LP problem instance in fixed MPS format from file `25fv47.mps`,[4] constructs an advanced initial basis, solves the instance with the primal simplex method (by default), and writes the solution to file `25fv47.txt`.

[4] This instance in fixed MPS format can be found in the Netlib LP collection; see `ftp://ftp.netlib.org/lp/data/`.

```
/* spxsamp1.c */

#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

int main(void)
{     glp_prob *P;
      P = glp_create_prob();
      glp_read_mps(P, GLP_MPS_DECK, NULL, "25fv47.mps");
      glp_adv_basis(P, NULL);
      glp_simplex(P, NULL);
      glp_print_sol(P, "25fv47.txt");
      glp_delete_prob(P);
      return 0;
}

/* eof */
```

Below here is shown the terminal output from this example program.

```
Reading problem data from '25fv47.mps'...
Problem: 25FV47
Objective: R0000
822 rows, 1571 columns, 11127 non-zeros
6919 records were read
Crashing...
Size of triangular part = 799
      0: obj =   1.627307307e+04  infeas =  5.194e+04 (23)
    200: obj =   1.474901610e+04  infeas =  1.233e+04 (19)
    400: obj =   1.343909995e+04  infeas =  3.648e+03 (13)
    600: obj =   1.756052217e+04  infeas =  4.179e+02 (7)
*   775: obj =   1.789251591e+04  infeas =  4.982e-14 (1)
*   800: obj =   1.663354510e+04  infeas =  2.857e-14 (1)
*  1000: obj =   1.024935068e+04  infeas =  1.958e-12 (1)
*  1200: obj =   7.860174791e+03  infeas =  2.810e-29 (1)
*  1400: obj =   6.642378184e+03  infeas =  2.036e-16 (1)
*  1600: obj =   6.037014568e+03  infeas =  0.000e+00 (1)
*  1800: obj =   5.662171307e+03  infeas =  6.447e-15 (1)
*  2000: obj =   5.528146165e+03  infeas =  9.764e-13 (1)
*  2125: obj =   5.501845888e+03  infeas =  0.000e+00 (1)
OPTIMAL SOLUTION FOUND
Writing basic solution to '25fv47.txt'...
```

Example 2

The following main program solves the same LP problem instance as in Example 1 above, however, it uses the dual simplex method, which starts from the standard initial basis.

```
/* spxsamp2.c */

#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

int main(void)
{     glp_prob *P;
      glp_smcp parm;
      P = glp_create_prob();
      glp_read_mps(P, GLP_MPS_DECK, NULL, "25fv47.mps");
      glp_init_smcp(&parm);
      parm.meth = GLP_DUAL;
      glp_simplex(P, &parm);
      glp_print_sol(P, "25fv47.txt");
      glp_delete_prob(P);
      return 0;
}

/* eof */
```

Below here is shown the terminal output from this example program.

```
Reading problem data from ‘25fv47.mps’...
Problem: 25FV47
Objective: R0000
822 rows, 1571 columns, 11127 non-zeros
6919 records were read
      0:                          infeas =  1.223e+03 (516)
    200:                          infeas =  7.000e+00 (471)
    240:                          infeas =  1.106e-14 (461)
|   400: obj =  -5.394267152e+03  infeas =  5.571e-16 (391)
|   600: obj =  -4.586395752e+03  infeas =  1.389e-15 (340)
|   800: obj =  -4.158268146e+03  infeas =  1.640e-15 (264)
|  1000: obj =  -3.725320045e+03  infeas =  5.181e-15 (245)
|  1200: obj =  -3.104802163e+03  infeas =  1.019e-14 (210)
|  1400: obj =  -2.584190499e+03  infeas =  8.865e-15 (178)
|  1600: obj =  -2.073852927e+03  infeas =  7.867e-15 (142)
|  1800: obj =  -1.164037407e+03  infeas =  8.792e-15 (109)
|  2000: obj =  -4.370590250e+02  infeas =  2.591e-14 (85)
|  2200: obj =   1.068240144e+03  infeas =  1.025e-13 (70)
|  2400: obj =   1.607481126e+03  infeas =  3.272e-14 (67)
```

```
|  2600: obj =   3.038230551e+03  infeas =  4.850e-14 (52)
|  2800: obj =   4.316238187e+03  infeas =  2.622e-14 (36)
|  3000: obj =   5.443842629e+03  infeas =  3.976e-15 (11)
|  3060: obj =   5.501845888e+03  infeas =  8.806e-15 (2)
OPTIMAL SOLUTION FOUND
Writing basic solution to '25fv47.txt'...
```

2.7.2 glp_exact—solve LP problem in exact arithmetic

Synopsis

```
int glp_exact(glp_prob *lp, const glp_smcp *parm);
```

Description

The routine `glp_exact` is a tentative implementation of the primal two-phase simplex method based on exact (rational) arithmetic. It is similar to the routine `glp_simplex`, however, for all internal computations it uses arithmetic of rational numbers, which is exact in mathematical sense, i.e. free of round-off errors unlike floating-point arithmetic.

Note that the routine `glp_exact` uses only two control parameters passed in the structure `glp_smcp`, namely, `it_lim` and `tm_lim`.

Returns

0	The LP problem instance has been successfully solved. (This code does *not* necessarily mean that the solver has found optimal solution. It only means that the solution process was successful.)
`GLP_EBADB`	Unable to start the search, because the initial basis specified in the problem object is invalid—the number of basic (auxiliary and structural) variables is not the same as the number of rows in the problem object.
`GLP_ESING`	Unable to start the search, because the basis matrix corresponding to the initial basis is exactly singular.
`GLP_EBOUND`	Unable to start the search, because some double-bounded (auxiliary or structural) variables have incorrect bounds.
`GLP_EFAIL`	The problem instance has no rows/columns.
`GLP_EITLIM`	The search was prematurely terminated, because the simplex iteration limit has been exceeded.
`GLP_ETMLIM`	The search was prematurely terminated, because the time limit has been exceeded.

Comments

Computations in exact arithmetic are very time consuming, so solving LP problem with the routine `glp_exact` from the very beginning is not a good idea. It is much better at first to find an optimal basis with the routine `glp_simplex` and only then to call `glp_exact`, in which case only a few simplex iterations need to be performed in exact arithmetic.

2.7.3 glp_init_smcp—initialize simplex solver control parameters

Synopsis

```
int glp_init_smcp(glp_smcp *parm);
```

Description

The routine `glp_init_smcp` initializes control parameters, which are used by the simplex solver, with default values.

Default values of the control parameters are stored in a `glp_smcp` structure, which the parameter `parm` points to.

2.7.4 glp_get_status—determine generic status of basic solution

Synopsis

```
int glp_get_status(glp_prob *lp);
```

Returns

The routine `glp_get_status` reports the generic status of the current basic solution for the specified problem object as follows:

`GLP_OPT`	solution is optimal;
`GLP_FEAS`	solution is feasible;
`GLP_INFEAS`	solution is infeasible;
`GLP_NOFEAS`	problem has no feasible solution;
`GLP_UNBND`	problem has unbounded solution;
`GLP_UNDEF`	solution is undefined.

More detailed information about the status of basic solution can be retrieved with the routines `glp_get_prim_stat` and `glp_get_dual_stat`.

2.7.5 glp_get_prim_stat—retrieve status of primal basic solution

Synopsis

```
int glp_get_prim_stat(glp_prob *lp);
```

Returns

The routine `glp_get_prim_stat` reports the status of the primal basic solution for the specified problem object as follows:

`GLP_UNDEF`	primal solution is undefined;
`GLP_FEAS`	primal solution is feasible;
`GLP_INFEAS`	primal solution is infeasible;
`GLP_NOFEAS`	no primal feasible solution exists.

2.7.6 glp_get_dual_stat—retrieve status of dual basic solution

Synopsis

```
int glp_get_dual_stat(glp_prob *lp);
```

Returns

The routine `glp_get_dual_stat` reports the status of the dual basic solution for the specified problem object as follows:

`GLP_UNDEF`	dual solution is undefined;
`GLP_FEAS`	dual solution is feasible;
`GLP_INFEAS`	dual solution is infeasible;
`GLP_NOFEAS`	no dual feasible solution exists.

2.7.7 glp_get_obj_val—retrieve objective value

Synopsis

```
double glp_get_obj_val(glp_prob *lp);
```

Returns

The routine `glp_get_obj_val` returns current value of the objective function.

2.7.8 glp_get_row_stat—retrieve row status

Synopsis

```
int glp_get_row_stat(glp_prob *lp, int i);
```

Returns

The routine `glp_get_row_stat` returns current status assigned to the auxiliary variable associated with `i`-th row as follows:

`GLP_BS`	basic variable;
`GLP_NL`	non-basic variable on its lower bound;
`GLP_NU`	non-basic variable on its upper bound;
`GLP_NF`	non-basic free (unbounded) variable;
`GLP_NS`	non-basic fixed variable.

2.7.9 glp_get_row_prim—retrieve row primal value

Synopsis

```
double glp_get_row_prim(glp_prob *lp, int i);
```

Returns

The routine `glp_get_row_prim` returns primal value of the auxiliary variable associated with `i`-th row.

2.7.10 glp_get_row_dual—retrieve row dual value

Synopsis

```
double glp_get_row_dual(glp_prob *lp, int i);
```

Returns

The routine `glp_get_row_dual` returns dual value (i.e. reduced cost) of the auxiliary variable associated with `i`-th row.

2.7.11 glp_get_col_stat—retrieve column status

Synopsis

```
int glp_get_col_stat(glp_prob *lp, int j);
```

Returns

The routine `glp_get_col_stat` returns current status assigned to the structural variable associated with `j`-th column as follows:

`GLP_BS`	basic variable;
`GLP_NL`	non-basic variable on its lower bound;
`GLP_NU`	non-basic variable on its upper bound;
`GLP_NF`	non-basic free (unbounded) variable;
`GLP_NS`	non-basic fixed variable.

2.7.12 glp_get_col_prim—retrieve column primal value

Synopsis

```
double glp_get_col_prim(glp_prob *lp, int j);
```

Returns

The routine `glp_get_col_prim` returns primal value of the structural variable associated with `j`-th column.

2.7.13 glp_get_col_dual—retrieve column dual value

Synopsis

```
double glp_get_col_dual(glp_prob *lp, int j);
```

Returns

The routine `glp_get_col_dual` returns dual value (i.e. reduced cost) of the structural variable associated with `j`-th column.

2.7.14 glp_get_unbnd_ray—determine variable causing unboundedness

Synopsis

```
int glp_get_unbnd_ray(glp_prob *lp);
```

Returns

The routine `glp_get_unbnd_ray` returns the number k of a variable, which causes primal or dual unboundedness. If $1 \leq k \leq m$, it is k-th auxiliary variable, and if $m + 1 \leq k \leq m + n$, it is $(k - m)$-th structural variable, where m is the number of rows, n is the number of columns in the problem object. If such variable is not defined, the routine returns 0.

Comments

If it is not exactly known which version of the simplex solver detected unboundedness, i.e. whether the unboundedness is primal or dual, it is sufficient to check the status of the variable with the routine `glp_get_row_stat` or `glp_get_col_stat`. If the variable is non-basic, the unboundedness is primal, otherwise, if the variable is basic, the unboundedness is dual (the latter case means that the problem has no primal feasible dolution).

2.8 Interior-point method routines

Interior-point methods (also known as *barrier methods*) are more modern and powerful numerical methods for large-scale linear programming. Such methods are especially efficient for very sparse LP problems and allow solving such problems much faster than the simplex method.

In brief, the GLPK interior-point solver works as follows.

At first, the solver transforms the original LP to a *working* LP in the standard format:

minimize

$$z = c_1 x_{m+1} + c_2 x_{m+2} + \ldots + c_n x_{m+n} + c_0 \tag{2.4}$$

subject to linear constraints

$$\begin{aligned} a_{11} x_{m+1} + a_{12} x_{m+2} + \ldots + a_{1n} x_{m+n} &= b_1 \\ a_{21} x_{m+1} + a_{22} x_{m+2} + \ldots + a_{2n} x_{m+n} &= b_2 \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots & \\ a_{m1} x_{m+1} + a_{m2} x_{m+2} + \ldots + a_{mn} x_{m+n} &= b_m \end{aligned} \tag{2.5}$$

and non-negative variables

$$x_1 \geq 0, \quad x_2 \geq 0, \quad \ldots, \quad x_n \geq 0 \tag{2.6}$$

where: z is the objective function; x_1, $\ldots$, x_n are variables; c_1, $\ldots$, c_n are objective coefficients; c_0 is a constant term of the objective function; a_{11}, $\ldots$, a_{mn} are constraint coefficients; b_1, $\ldots$, b_m are right-hand sides.

Using vector and matrix notations the working LP (2.4)—(2.6) can be written as follows:

$$z = c^T x + c_0 \ \rightarrow \ \min, \tag{2.7}$$

$$Ax = b, \tag{2.8}$$

$$x \geq 0, \tag{2.9}$$

where: $x = (x_j)$ is n-vector of variables, $c = (c_j)$ is n-vector of objective coefficients, $A = (a_{ij})$ is $m \times n$-matrix of constraint coefficients, and $b = (b_i)$ is m-vector of right-hand sides.

Karush–Kuhn–Tucker optimality conditions for LP (2.7)—(2.9) are the following:

$$Ax = b, \tag{2.10}$$

$$A^T\pi + \lambda = c, \tag{2.11}$$

$$\lambda^T x = 0, \tag{2.12}$$

$$x \geq 0, \quad \lambda \geq 0, \tag{2.13}$$

where: π is m-vector of Lagrange multipliers (dual variables) for equality constraints (2.8), λ is n-vector of Lagrange multipliers (dual variables) for non-negativity constraints (2.9), (2.10) is the primal feasibility condition, (2.11) is the dual feasibility condition, (2.12) is the primal-dual complementarity condition, and (2.13) is the non-negativity conditions.

The main idea of the primal-dual interior-point method is based on finding a point in the primal-dual space (i.e. in the space of all primal and dual variables x, π, and λ), which satisfies to all optimality conditions (2.10)—(2.13). Obviously, x-component of such point then provides an optimal solution to the working LP (2.7)—(2.9).

To find the optimal point (x^*, π^*, λ^*) the interior-point method attempts to solve the system of equations (2.10)—(2.12), which is closed in the sense that the number of variables x_j, π_i, and λ_j and the number equations are the same and equal to $m + 2n$. Due to condition (2.12) this system of equations is non-linear, so it can be solved with a version of *Newton's method* provided with additional rules to keep the current point within the positive orthant as required by the non-negativity conditions (2.13).

Finally, once the optimal point (x^*, π^*, λ^*) has been found, the solver performs inverse transformations to recover corresponding solution to the original LP passed to the solver from the application program.

2.8.1 glp_interior—solve LP problem with the interior-point method

Synopsis

```
int glp_interior(glp_prob *P, const glp_iptcp *parm);
```

Description

The routine `glp_interior` is a driver to the LP solver based on the primal-dual interior-point method. This routine retrieves problem data from the specified problem object, calls the solver to solve the problem instance, and stores results of computations back into the problem object.

The interior-point solver has a set of control parameters. Values of the control parameters can be passed in the structure `glp_iptcp`, which the parameter `parm` points to. For detailed description of this structure see paragraph "Control parameters" below. Before specifying some control parameters the application program should initialize the structure `glp_iptcp` by default values of all control parameters using the routine `glp_init_iptcp` (see the next subsection). This is needed for backward compatibility, because in the future there may appear new members in the structure `glp_iptcp`.

The parameter `parm` can be specified as `NULL`, in which case the solver uses default settings.

Returns

0	The LP problem instance has been successfully solved. (This code does *not* necessarily mean that the solver has found optimal solution. It only means that the solution process was successful.)
`GLP_EFAIL`	The problem has no rows/columns.
`GLP_ENOCVG`	Very slow convergence or divergence.
`GLP_EITLIM`	Iteration limit exceeded.
`GLP_EINSTAB`	Numerical instability on solving Newtonian system.

Comments

The routine `glp_interior` implements an easy version of the primal-dual interior-point method based on Mehrotra's technique.[5]

Note that currently the GLPK interior-point solver does not include many important features, in particular:

• it is not able to process dense columns. Thus, if the constraint matrix of the LP problem has dense columns, the solving process may be inefficient;

• it has no features against numerical instability. For some LP problems premature termination may happen if the matrix ADA^T becomes singular or ill-conditioned;

• it is not able to identify the optimal basis, which corresponds to the interior-point solution found.

[5] S. Mehrotra. On the implementation of a primal-dual interior point method. SIAM J. on Optim., 2(4), pp. 575-601, 1992.

Terminal output

Solving large LP problems may take a long time, so the solver reports some information about every interior-point iteration,[6] which is sent to the terminal. This information has the following format:

```
nnn: F = fff; rpi = ppp; rdi = ddd; gap = ggg
```

where: `nnn` is iteration number, `fff` is the current value of the objective function (in the case of maximization it has wrong sign), `ppp` is the current relative primal infeasibility (cf. (2.10)):

$$\frac{\|Ax^{(k)} - b\|}{1 + \|b\|}, \tag{2.14}$$

`ddd` is the current relative dual infeasibility (cf. (2.11)):

$$\frac{\|A^T\pi^{(k)} + \lambda^{(k)} - c\|}{1 + \|c\|}, \tag{2.15}$$

`ggg` is the current primal-dual gap (cf. (2.12)):

$$\frac{|c^Tx^{(k)} - b^T\pi^{(k)}|}{1 + |c^Tx^{(k)}|}, \tag{2.16}$$

and $[x^{(k)}, \pi^{(k)}, \lambda^{(k)}]$ is the current point on k-th iteration, $k = 0, 1, 2, \ldots$. Note that all solution components are internally scaled, so information sent to the terminal is suitable only for visual inspection.

Control parameters

This paragraph describes all control parameters currently used in the interior-point solver. Symbolic names of control parameters are names of corresponding members in the structure `glp_iptcp`.

`int msg_lev` (default: `GLP_MSG_ALL`)
Message level for terminal output:
`GLP_MSG_OFF`—no output;
`GLP_MSG_ERR`—error and warning messages only;
`GLP_MSG_ON` —normal output;
`GLP_MSG_ALL`—full output (including informational messages).

[6] Unlike the simplex method the interior point method usually needs 30—50 iterations (independently on the problem size) in order to find an optimal solution.

`int ord_alg` (default: `GLP_ORD_AMD`)
Ordering algorithm used prior to Cholesky factorization:
`GLP_ORD_NONE` —use natural (original) ordering;
`GLP_ORD_QMD` —quotient minimum degree (QMD);
`GLP_ORD_AMD` —approximate minimum degree (AMD);
`GLP_ORD_SYMAMD`—approximate minimum degree (SYMAMD).

Example

The following main program reads LP problem instance in fixed MPS format from file `25fv47.mps`,[7] solves it with the interior-point solver, and writes the solution to file `25fv47.txt`.

```
/* iptsamp.c */

#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

int main(void)
{     glp_prob *P;
      P = glp_create_prob();
      glp_read_mps(P, GLP_MPS_DECK, NULL, "25fv47.mps");
      glp_interior(P, NULL);
      glp_print_ipt(P, "25fv47.txt");
      glp_delete_prob(P);
      return 0;
}

/* eof */
```

Below here is shown the terminal output from this example program.

```
Reading problem data from '25fv47.mps'...
Problem: 25FV47
Objective: R0000
822 rows, 1571 columns, 11127 non-zeros
6919 records were read
Original LP has 822 row(s), 1571 column(s), and 11127 non-zero(s)
Working LP has 821 row(s), 1876 column(s), and 10705 non-zero(s)
Matrix A has 10705 non-zeros
Matrix S = A*A' has 11895 non-zeros (upper triangle)
Minimal degree ordering...
Computing Cholesky factorization S = L'*L...
```

[7] This instance in fixed MPS format can be found in the Netlib LP collection; see `ftp://ftp.netlib.org/lp/data/`.

```
Matrix L has 35411 non-zeros
Guessing initial point...
Optimization begins...
  0: obj =   1.823377629e+05; rpi =  1.3e+01; rdi =  1.4e+01; gap =  9.3e-01
  1: obj =   9.260045192e+04; rpi =  5.3e+00; rdi =  5.6e+00; gap =  6.8e+00
  2: obj =   3.596999742e+04; rpi =  1.5e+00; rdi =  1.2e+00; gap =  1.8e+01
  3: obj =   1.989627568e+04; rpi =  4.7e-01; rdi =  3.0e-01; gap =  1.9e+01
  4: obj =   1.430215557e+04; rpi =  1.1e-01; rdi =  8.6e-02; gap =  1.4e+01
  5: obj =   1.155716505e+04; rpi =  2.3e-02; rdi =  2.4e-02; gap =  6.8e+00
  6: obj =   9.660273208e+03; rpi =  6.7e-03; rdi =  4.6e-03; gap =  3.9e+00
  7: obj =   8.694348283e+03; rpi =  3.7e-03; rdi =  1.7e-03; gap =  2.0e+00
  8: obj =   8.019543639e+03; rpi =  2.4e-03; rdi =  3.9e-04; gap =  1.0e+00
  9: obj =   7.122676293e+03; rpi =  1.2e-03; rdi =  1.5e-04; gap =  6.6e-01
 10: obj =   6.514534518e+03; rpi =  6.1e-04; rdi =  4.3e-05; gap =  4.1e-01
 11: obj =   6.361572203e+03; rpi =  4.8e-04; rdi =  2.2e-05; gap =  3.0e-01
 12: obj =   6.203355508e+03; rpi =  3.2e-04; rdi =  1.7e-05; gap =  2.6e-01
 13: obj =   6.032943411e+03; rpi =  2.0e-04; rdi =  9.3e-06; gap =  2.1e-01
 14: obj =   5.796553021e+03; rpi =  9.8e-05; rdi =  3.2e-06; gap =  1.0e-01
 15: obj =   5.667032431e+03; rpi =  4.4e-05; rdi =  1.1e-06; gap =  5.6e-02
 16: obj =   5.613911867e+03; rpi =  2.5e-05; rdi =  4.1e-07; gap =  3.5e-02
 17: obj =   5.560572626e+03; rpi =  9.9e-06; rdi =  2.3e-07; gap =  2.1e-02
 18: obj =   5.537276001e+03; rpi =  5.5e-06; rdi =  8.4e-08; gap =  1.1e-02
 19: obj =   5.522746942e+03; rpi =  2.2e-06; rdi =  4.0e-08; gap =  6.7e-03
 20: obj =   5.509956679e+03; rpi =  7.5e-07; rdi =  1.8e-08; gap =  2.9e-03
 21: obj =   5.504571733e+03; rpi =  1.6e-07; rdi =  5.8e-09; gap =  1.1e-03
 22: obj =   5.502576367e+03; rpi =  3.4e-08; rdi =  1.0e-09; gap =  2.5e-04
 23: obj =   5.502057119e+03; rpi =  8.1e-09; rdi =  3.0e-10; gap =  7.7e-05
 24: obj =   5.501885996e+03; rpi =  9.4e-10; rdi =  1.2e-10; gap =  2.4e-05
 25: obj =   5.501852464e+03; rpi =  1.4e-10; rdi =  1.2e-11; gap =  3.0e-06
 26: obj =   5.501846549e+03; rpi =  1.4e-11; rdi =  1.2e-12; gap =  3.0e-07
 27: obj =   5.501845954e+03; rpi =  1.4e-12; rdi =  1.2e-13; gap =  3.0e-08
 28: obj =   5.501845895e+03; rpi =  1.5e-13; rdi =  1.2e-14; gap =  3.0e-09
OPTIMAL SOLUTION FOUND
Writing interior-point solution to '25fv47.txt'...
```

2.8.2 glp_init_iptcp—initialize interior-point solver control parameters

Synopsis

```
int glp_init_iptcp(glp_iptcp *parm);
```

Description

The routine `glp_init_iptcp` initializes control parameters, which are used by the interior-point solver, with default values.

Default values of the control parameters are stored in the structure `glp_iptcp`, which the parameter `parm` points to.

2.8.3 glp_ipt_status—determine solution status

Synopsis

```
int glp_ipt_status(glp_prob *lp);
```

Returns

The routine `glp_ipt_status` reports the status of a solution found by the interior-point solver as follows:

`GLP_UNDEF`	interior-point solution is undefined.
`GLP_OPT`	interior-point solution is optimal.
`GLP_INFEAS`	interior-point solution is infeasible.
`GLP_NOFEAS`	no feasible primal-dual solution exists.

2.8.4 glp_ipt_obj_val—retrieve objective value

Synopsis

```
double glp_ipt_obj_val(glp_prob *lp);
```

Returns

The routine `glp_ipt_obj_val` returns value of the objective function for interior-point solution.

2.8.5 glp_ipt_row_prim—retrieve row primal value

Synopsis

```
double glp_ipt_row_prim(glp_prob *lp, int i);
```

Returns

The routine `glp_ipt_row_prim` returns primal value of the auxiliary variable associated with `i`-th row.

2.8.6 glp_ipt_row_dual—retrieve row dual value

Synopsis

```
double glp_ipt_row_dual(glp_prob *lp, int i);
```

Returns

The routine `glp_ipt_row_dual` returns dual value (i.e. reduced cost) of the auxiliary variable associated with `i`-th row.

2.8.7 glp_ipt_col_prim—retrieve column primal value

Synopsis

```
double glp_ipt_col_prim(glp_prob *lp, int j);
```

Returns

The routine `glp_ipt_col_prim` returns primal value of the structural variable associated with `j`-th column.

2.8.8 glp_ipt_col_dual—retrieve column dual value

Synopsis

```
double glp_ipt_col_dual(glp_prob *lp, int j);
```

Returns

The routine `glp_ipt_col_dual` returns dual value (i.e. reduced cost) of the structural variable associated with `j`-th column.

2.9 Mixed integer programming routines

2.9.1 glp_set_col_kind—set (change) column kind

Synopsis

```
void glp_set_col_kind(glp_prob *mip, int j, int kind);
```

Description

The routine `glp_set_col_kind` sets (changes) the kind of `j`-th column (structural variable) as specified by the parameter `kind`:

`GLP_CV` continuous variable;
`GLP_IV` integer variable;
`GLP_BV` binary variable.

Setting a column to `GLP_BV` has the same effect as if it were set to `GLP_IV`, its lower bound were set 0, and its upper bound were set to 1.

2.9.2 glp_get_col_kind—retrieve column kind

Synopsis

```
int glp_get_col_kind(glp_prob *mip, int j);
```

Returns

The routine `glp_get_col_kind` returns the kind of `j`-th column (structural variable) as follows:

`GLP_CV` continuous variable;
`GLP_IV` integer variable;
`GLP_BV` binary variable.

2.9.3 glp_get_num_int—retrieve number of integer columns

Synopsis

```
int glp_get_num_int(glp_prob *mip);
```

Returns

The routine `glp_get_num_int` returns the number of columns (structural variables), which are marked as integer. Note that this number *does* include binary columns.

2.9.4 glp_get_num_bin—retrieve number of binary columns

Synopsis

```
int glp_get_num_bin(glp_prob *mip);
```

Returns

The routine `glp_get_num_bin` returns the number of columns (structural variables), which are marked as integer and whose lower bound is zero and upper bound is one.

2.9.5 glp_intopt—solve MIP problem with the branch-and-cut method

Synopsis

```
int glp_intopt(glp_prob *mip, const glp_iocp *parm);
```

Description

The routine `glp_intopt` is a driver to the MIP solver based on the branch-and-cut method, which is a hybrid of branch-and-bound and cutting plane methods.

If the presolver is disabled (see paragraph "Control parameters" below), on entry to the routine `glp_intopt` the problem object, which the parameter `mip` points to, should contain optimal solution to LP relaxation (it can be obtained, for example, with the routine `glp_simplex`). Otherwise, if the presolver is enabled, it is not necessary.

The MIP solver has a set of control parameters. Values of the control parameters can be passed in the structure `glp_iocp`, which the parameter `parm` points to. For detailed description of this structure see paragraph "Control parameters" below. Before specifying some control parameters the application program should initialize the structure `glp_iocp` by default values of all control parameters using the routine `glp_init_iocp` (see the next subsection). This is needed for backward compatibility, because in the future there may appear new members in the structure `glp_iocp`.

The parameter `parm` can be specified as `NULL`, in which case the solver uses default settings.

Note that the GLPK branch-and-cut solver is not perfect, so it is unable to solve hard or very large scale MIP instances for a reasonable time.

Returns

0	The MIP problem instance has been successfully solved. (This code does *not* necessarily mean that the solver has found optimal solution. It only means that the solution process was successful.)
`GLP_EBOUND`	Unable to start the search, because some double-bounded variables have incorrect bounds or some integer variables have non-integer (fractional) bounds.
`GLP_EROOT`	Unable to start the search, because optimal basis for initial LP relaxation is not provided. (This code may appear only if the presolver is disabled.)
`GLP_ENOPFS`	Unable to start the search, because LP relaxation of the MIP problem instance has no primal feasible solution. (This code may appear only if the presolver is enabled.)
`GLP_ENODFS`	Unable to start the search, because LP relaxation of the MIP problem instance has no dual feasible solution. In other word, this code means that if the LP relaxation has at least one primal feasible solution, its optimal solution is unbounded, so if the MIP problem has at least one integer feasible solution, its (integer) optimal solution is also unbounded. (This code may appear only if the presolver is enabled.)
`GLP_EFAIL`	The search was prematurely terminated due to the solver failure.
`GLP_EMIPGAP`	The search was prematurely terminated, because the relative mip gap tolerance has been reached.
`GLP_ETMLIM`	The search was prematurely terminated, because the time limit has been exceeded.
`GLP_ESTOP`	The search was prematurely terminated by application. (This code may appear only if the advanced solver interface is used.)

Built-in MIP presolver

The branch-and-cut solver has *built-in MIP presolver*. It is a subprogram that transforms the original MIP problem specified in the problem object to an equivalent MIP problem, which may be easier for solving with the branch-and-cut method than the original one. For example, the presolver can remove redundant constraints and variables, whose optimal values are known, perform bound and coefficient reduction, etc. Once the transformed

MIP problem has been solved, the presolver transforms its solution back to corresponding solution of the original problem.

Presolving is an optional feature of the routine `glp_intopt`, and by default it is disabled. In order to enable the MIP presolver, the control parameter `presolve` should be set to `GLP_ON` (see paragraph "Control parameters" below).

Advanced solver interface

The routine `glp_intopt` allows the user to control the branch-and-cut search by passing to the solver a user-defined callback routine. For more details see Chapter "Branch-and-Cut API Routines".

Terminal output

Solving a MIP problem may take a long time, so the solver reports some information about best known solutions, which is sent to the terminal. This information has the following format:

```
+nnn: mip = xxx <rho> yyy gap (ppp; qqq)
```

where: '`nnn`' is the simplex iteration number; '`xxx`' is a value of the objective function for the best known integer feasible solution (if no integer feasible solution has been found yet, '`xxx`' is the text '`not found yet`'); '`rho`' is the string '`>=`' (in case of minimization) or '`<=`' (in case of maximization); '`yyy`' is a global bound for exact integer optimum (i.e. the exact integer optimum is always in the range from '`xxx`' to '`yyy`'); '`gap`' is the relative mip gap, in percents, computed as $gap = |xxx - yyy|/(|xxx| + \texttt{DBL\_EPSILON}) \cdot 100\%$ (if *gap* is greater than 999.9%, it is not printed); '`ppp`' is the number of subproblems in the active list, '`qqq`' is the number of subproblems which have been already fathomed and therefore removed from the branch-and-bound search tree.

Control parameters

This paragraph describes all control parameters currently used in the MIP solver. Symbolic names of control parameters are names of corresponding members in the structure `glp_iocp`.

`int msg_lev` (default: `GLP_MSG_ALL`)
Message level for terminal output:
`GLP_MSG_OFF`—no output;
`GLP_MSG_ERR`—error and warning messages only;
`GLP_MSG_ON` —normal output;
`GLP_MSG_ALL`—full output (including informational messages).

`int br_tech` (default: `GLP_BR_DTH`)
Branching technique option:
`GLP_BR_FFV`—first fractional variable;
`GLP_BR_LFV`—last fractional variable;
`GLP_BR_MFV`—most fractional variable;
`GLP_BR_DTH`—heuristic by Driebeck and Tomlin;
`GLP_BR_PCH`—hybrid pseudocost heuristic.

`int bt_tech` (default: `GLP_BT_BLB`)
Backtracking technique option:
`GLP_BT_DFS`—depth first search;
`GLP_BT_BFS`—breadth first search;
`GLP_BT_BLB`—best local bound;
`GLP_BT_BPH`—best projection heuristic.

`int pp_tech` (default: `GLP_PP_ALL`)
Preprocessing technique option:
`GLP_PP_NONE`—disable preprocessing;
`GLP_PP_ROOT`—perform preprocessing only on the root level;
`GLP_PP_ALL` —perform preprocessing on all levels.

`int fp_heur` (default: `GLP_OFF`)
Feasibility pump heuristic option:
`GLP_ON` —enable applying the feasibility pump heuristic;
`GLP_OFF`—disable applying the feasibility pump heuristic.

`int gmi_cuts` (default: `GLP_OFF`)
Gomory's mixed integer cut option:
`GLP_ON` —enable generating Gomory's cuts;
`GLP_OFF`—disable generating Gomory's cuts.

`int mir_cuts` (default: `GLP_OFF`)
Mixed integer rounding (MIR) cut option:
`GLP_ON` —enable generating MIR cuts;
`GLP_OFF`—disable generating MIR cuts.

`int cov_cuts` (default: `GLP_OFF`)
Mixed cover cut option:
`GLP_ON` —enable generating mixed cover cuts;
`GLP_OFF`—disable generating mixed cover cuts.

`int clq_cuts` (default: `GLP_OFF`)
Clique cut option:
`GLP_ON` —enable generating clique cuts;
`GLP_OFF`—disable generating clique cuts.

`double tol_int` (default: `1e-5`)
Absolute tolerance used to check if optimal solution to the current LP relaxation is integer feasible. (Do not change this parameter without detailed understanding its purpose.)

`double tol_obj` (default: `1e-7`)
Relative tolerance used to check if the objective value in optimal solution to the current LP relaxation is not better than in the best known integer feasible solution. (Do not change this parameter without detailed understanding its purpose.)

`double mip_gap` (default: `0.0`)
The relative mip gap tolerance. If the relative mip gap for currently known best integer feasible solution falls below this tolerance, the solver terminates the search. This allows obtainig suboptimal integer feasible solutions if solving the problem to optimality takes too long time.

`int tm_lim` (default: `INT_MAX`)
Searching time limit, in milliseconds.

`int out_frq` (default: `5000`)
Output frequency, in milliseconds. This parameter specifies how frequently the solver sends information about the solution process to the terminal.

`int out_dly` (default: `10000`)
Output delay, in milliseconds. This parameter specifies how long the solver should delay sending information about solution of the current LP relaxation with the simplex method to the terminal.

`void (*cb_func)(glp_tree *tree, void *info)` (default: `NULL`)
Entry point to the user-defined callback routine. `NULL` means the advanced solver interface is not used. For more details see Chapter "Branch-and-Cut API Routines".

`void *cb_info` (default: `NULL`)
Transit pointer passed to the routine `cb_func` (see above).

`int cb_size` (default: `0`)
The number of extra (up to 256) bytes allocated for each node of the branch-and-bound tree to store application-specific data. On creating a node these bytes are initialized by binary zeros.

`int presolve` (default: `GLP_OFF`)
MIP presolver option:
`GLP_ON` —enable using the MIP presolver;
`GLP_OFF`—disable using the MIP presolver.

`int binarize` (default: `GLP_OFF`)
Binarization option (used only if the presolver is enabled):
`GLP_ON` —replace general integer variables by binary ones;
`GLP_OFF`—do not use binarization.

2.9.6 glp_init_iocp—initialize integer optimizer control parameters

Synopsis

```
void glp_init_iocp(glp_iocp *parm);
```

Description

The routine `glp_init_iocp` initializes control parameters, which are used by the branch-and-cut solver, with default values.

Default values of the control parameters are stored in a `glp_iocp` structure, which the parameter `parm` points to.

2.9.7 glp_mip_status—determine status of MIP solution

Synopsis

```
int glp_mip_status(glp_prob *mip);
```

Returns

The routine `glp_mip_status` reports the status of a MIP solution found by the MIP solver as follows:

GLP_UNDEF	MIP solution is undefined.
GLP_OPT	MIP solution is integer optimal.
GLP_FEAS	MIP solution is integer feasible, however, its optimality (or non-optimality) has not been proven, perhaps due to premature termination of the search.
GLP_NOFEAS	problem has no integer feasible solution (proven by the solver).

2.9.8 glp_mip_obj_val—retrieve objective value

Synopsis

```
double glp_mip_obj_val(glp_prob *mip);
```

Returns

The routine glp_mip_obj_val returns value of the objective function for MIP solution.

2.9.9 glp_mip_row_val—retrieve row value

Synopsis

```
double glp_mip_row_val(glp_prob *mip, int i);
```

Returns

The routine glp_mip_row_val returns value of the auxiliary variable associated with i-th row for MIP solution.

2.9.10 glp_mip_col_val—retrieve column value

Synopsis

```
double glp_mip_col_val(glp_prob *mip, int j);
```

Returns

The routine glp_mip_col_val returns value of the structural variable associated with j-th column for MIP solution.

2.10 Additional routines

2.10.1 lpx_check_kkt—check Karush-Kuhn-Tucker optimality conditions

Synopsis

```
void lpx_check_kkt(glp_prob *lp, int scaled, LPXKKT *kkt);
```

Description

The routine `lpx_check_kkt` checks Karush-Kuhn-Tucker optimality conditions for basic solution. It is assumed that both primal and dual components of basic solution are valid.

If the parameter `scaled` is zero, the optimality conditions are checked for the original, unscaled LP problem. Otherwise, if the parameter `scaled` is non-zero, the routine checks the conditions for an internally scaled LP problem.

The parameter `kkt` is a pointer to the structure `LPXKKT`, to which the routine stores results of the check. Members of this structure are shown in the table below.

The routine performs all computations using only components of the given LP problem and the current basic solution.

Background

The first condition checked by the routine is:

$$x_R - Ax_S = 0, \tag{KKT.PE}$$

where x_R is the subvector of auxiliary variables (rows), x_S is the subvector of structural variables (columns), A is the constraint matrix. This condition expresses the requirement that all primal variables must satisfy to the system of equality constraints of the original LP problem. In case of exact arithmetic this condition would be satisfied for any basic solution; however, in case of inexact (floating-point) arithmetic, this condition shows how accurate the primal basic solution is, that depends on accuracy of a representation of the basis matrix used by the simplex method routines.

The second condition checked by the routine is:

$$l_k \le x_k \le u_k \quad \text{for all } k = 1, \ldots, m + n, \tag{KKT.PB}$$

Condition	Member	Comment
(KKT.PE)	`pe_ae_max`	Largest absolute error
	`pe_ae_row`	Number of row with largest absolute error
	`pe_re_max`	Largest relative error
	`pe_re_row`	Number of row with largest relative error
	`pe_quality`	Quality of primal solution
(KKT.PB)	`pb_ae_max`	Largest absolute error
	`pb_ae_ind`	Number of variable with largest absolute error
	`pb_re_max`	Largest relative error
	`pb_re_ind`	Number of variable with largest relative error
	`pb_quality`	Quality of primal feasibility
(KKT.DE)	`de_ae_max`	Largest absolute error
	`de_ae_col`	Number of column with largest absolute error
	`de_re_max`	Largest relative error
	`de_re_col`	Number of column with largest relative error
	`de_quality`	Quality of dual solution
(KKT.DB)	`db_ae_max`	Largest absolute error
	`db_ae_ind`	Number of variable with largest absolute error
	`db_re_max`	Largest relative error
	`db_re_ind`	Number of variable with largest relative error
	`db_quality`	Quality of dual feasibility

where x_k is auxiliary ($1 \le k \le m$) or structural ($m+1 \le k \le m+n$) variable, l_k and u_k are, respectively, lower and upper bounds of the variable x_k (including cases of infinite bounds). This condition expresses the requirement that all primal variables must satisfy to bound constraints of the original LP problem. Since in case of basic solution all non-basic variables are placed on their bounds, actually the condition (KKT.PB) needs to be checked for basic variables only. If the primal basic solution has sufficient accuracy, this condition shows primal feasibility of the solution.

The third condition checked by the routine is:

$$\text{grad } Z = c = (\tilde{A})^T \pi + d,$$

where Z is the objective function, c is the vector of objective coefficients, $(\tilde{A})^T$ is a matrix transposed to the expanded constraint matrix $\tilde{A} = (I|-A)$, π is a vector of Lagrange multipliers that correspond to equality constraints of the original LP problem, d is a vector of Lagrange multipliers that correspond to bound constraints for all (auxiliary and structural) variables of

the original LP problem. Geometrically the third condition expresses the requirement that the gradient of the objective function must belong to the orthogonal complement of a linear subspace defined by the equality and active bound constraints, i.e. that the gradient must be a linear combination of normals to the constraint planes, where Lagrange multipliers π and d are coefficients of that linear combination.

To eliminate the vector π the third condition can be rewritten as:

$$\begin{pmatrix} I \\ -A^T \end{pmatrix} \pi = \begin{pmatrix} d_R \\ d_S \end{pmatrix} + \begin{pmatrix} c_R \\ c_S \end{pmatrix},$$

or, equivalently:

$$\begin{aligned} \pi + d_R &= c_R, \\ -A^T\pi + d_S &= c_S. \end{aligned}$$

Then substituting the vector π from the first equation into the second one we have:

$$A^T(d_R - c_R) + (d_S - c_S) = 0, \qquad \text{(KKT.DE)}$$

where d_R is the subvector of reduced costs of auxiliary variables (rows), d_S is the subvector of reduced costs of structural variables (columns), c_R and c_S are subvectors of objective coefficients at, respectively, auxiliary and structural variables, A^T is a matrix transposed to the constraint matrix of the original LP problem. In case of exact arithmetic this condition would be satisfied for any basic solution; however, in case of inexact (floating-point) arithmetic, this condition shows how accurate the dual basic solution is, that depends on accuracy of a representation of the basis matrix used by the simplex method routines.

The last, fourth condition checked by the routine is (KKT.DB):

$d_k = 0,$	if x_k is basic or free non-basic variable
$0 \le d_k < +\infty$	if x_k is non-basic on its lower (minimization) or upper (maximization) bound
$-\infty < d_k \le 0$	if x_k is non-basic on its upper (minimization) or lower (maximization) bound
$-\infty < d_k < +\infty$	if x_k is non-basic fixed variable

for all $k = 1, \ldots, m + n$, where d_k is a reduced cost (Lagrange multiplier) of auxiliary ($1 \le k \le m$) or structural ($m + 1 \le k \le m + n$) variable x_k. Geometrically this condition expresses the requirement that constraints of the original problem must "hold" the point preventing its movement along the anti-gradient (in case of minimization) or the gradient (in case of maximization) of the objective function. Since in case of basic solution reduced

costs of all basic variables are placed on their (zero) bounds, actually the condition (KKT.DB) needs to be checked for non-basic variables only. If the dual basic solution has sufficient accuracy, this condition shows dual feasibility of the solution.

Should note that the complete set of Karush-Kuhn-Tucker optimality conditions also includes the fifth, so called complementary slackness condition, which expresses the requirement that at least either a primal variable x_k or its dual counterpart d_k must be on its bound for all $k = 1, \ldots, m + n$. However, being always satisfied by definition for any basic solution that condition is not checked by the routine.

To check the first condition (KKT.PE) the routine computes a vector of residuals:

$$g = x_R - Ax_S,$$

determines component of this vector that correspond to largest absolute and relative errors:

$$\texttt{pe\_ae\_max} = \max_{1 \le i \le m} |g_i|,$$

$$\texttt{pe\_re\_max} = \max_{1 \le i \le m} \frac{|g_i|}{1 + |(x_R)_i|},$$

and stores these quantities and corresponding row indices to the structure `LPXKKT`.

To check the second condition (KKT.PB) the routine computes a vector of residuals:

$$h_k = \begin{cases} 0, & \text{if } l_k \le x_k \le u_k \\ x_k - l_k, & \text{if } x_k < l_k \\ x_k - u_k, & \text{if } x_k > u_k \end{cases}$$

for all $k = 1, \ldots, m + n$, determines components of this vector that correspond to largest absolute and relative errors:

$$\texttt{pb\_ae\_max} = \max_{1 \le k \le m+n} |h_k|,$$

$$\texttt{pb\_re\_max} = \max_{1 \le k \le m+n} \frac{|h_k|}{1 + |x_k|},$$

and stores these quantities and corresponding variable indices to the structure `LPXKKT`.

To check the third condition (KKT.DE) the routine computes a vector of residuals:

$$u = A^T(d_R - c_R) + (d_S - c_S),$$

determines components of this vector that correspond to largest absolute and relative errors:

$$\texttt{de\_ae\_max} = \max_{1 \le j \le n} |u_j|,$$

$$\texttt{de\_re\_max} = \max_{1 \le j \le n} \frac{|u_j|}{1 + |(d_S)_j - (c_S)_j|},$$

and stores these quantities and corresponding column indices to the structure `LPXKKT`.

To check the fourth condition (KKT.DB) the routine computes a vector of residuals:

$$v_k = \begin{cases} 0, & \text{if } d_k \text{ has correct sign} \\ d_k, & \text{if } d_k \text{ has wrong sign} \end{cases}$$

for all $k = 1, \ldots, m + n$, determines components of this vector that correspond to largest absolute and relative errors:

$$\texttt{db\_ae\_max} = \max_{1 \le k \le m+n} |v_k|,$$

$$\texttt{db\_re\_max} = \max_{1 \le k \le m+n} \frac{|v_k|}{1 + |d_k - c_k|},$$

and stores these quantities and corresponding variable indices to the structure `LPXKKT`.

Using the relative errors for all the four conditions listed above the routine `lpx_check_kkt` also estimates a "quality" of the basic solution from the standpoint of these conditions and stores corresponding quality indicators to the structure `LPXKKT`:

`pe_quality`—quality of primal solution;
`pb_quality`—quality of primal feasibility;
`de_quality`—quality of dual solution;
`db_quality`—quality of dual feasibility.

Each of these indicators is assigned to one of the following four values:
`'H'` means high quality,
`'M'` means medium quality,
`'L'` means low quality, or
`'?'` means wrong or infeasible solution.

If all the indicators show high or medium quality (for an internally scaled LP problem, i.e. when the parameter `scaled` in a call to the routine `lpx_check_kkt` is non-zero), the user can be sure that the obtained basic solution is quite accurate.

If some of the indicators show low quality, the solution can still be considered as relevant, though an additional analysis is needed depending on which indicator shows low quality.

If the indicator `pe_quality` is assigned to `'?'`, the primal solution is wrong. If the indicator `de_quality` is assigned to `'?'`, the dual solution is wrong.

If the indicator `db_quality` is assigned to `'?'` while other indicators show a good quality, this means that the current basic solution being primal feasible is not dual feasible. Similarly, if the indicator `pb_quality` is assigned to `'?'` while other indicators are not, this means that the current basic solution being dual feasible is not primal feasible.

Chapter 3

Utility API routines

3.1 Problem data reading/writing routines

3.1.1 glp_read_mps—read problem data in MPS format

Synopsis

```
int glp_read_mps(glp_prob *lp, int fmt, const void *parm,
      const char *fname);
```

Description

The routine `glp_read_mps` reads problem data in MPS format from a text file. (The MPS format is described in Appendix B, page 236.)

The parameter `fmt` specifies the MPS format version as follows:

`GLP_MPS_DECK` fixed (ancient) MPS format;

`GLP_MPS_FILE` free (modern) MPS format.

The parameter `parm` is reserved for use in the future and must be specified as `NULL`.

The character string `fname` specifies a name of the text file to be read in. (If the file name ends with suffix '`.gz`', the file is assumed to be compressed, in which case the routine `glp_read_mps` decompresses it "on the fly".)

Note that before reading data the current content of the problem object is completely erased with the routine `glp_erase_prob`.

Returns

If the operation was successful, the routine `glp_read_mps` returns zero. Otherwise, it prints an error message and returns non-zero.

3.1.2 glp_write_mps—write problem data in MPS format

Synopsis

```
int glp_write_mps(glp_prob *lp, int fmt, const void *parm,
      const char *fname);
```

Description

The routine `glp_write_mps` writes problem data in MPS format to a text file. (The MPS format is described in Appendix B, page 236.)

The parameter `fmt` specifies the MPS format version as follows:

`GLP_MPS_DECK` fixed (ancient) MPS format;

`GLP_MPS_FILE` free (modern) MPS format.

The parameter `parm` is reserved for use in the future and must be specified as `NULL`.

The character string `fname` specifies a name of the text file to be written out. (If the file name ends with suffix '`.gz`', the file is assumed to be compressed, in which case the routine `glp_write_mps` performs automatic compression on writing it.)

Returns

If the operation was successful, the routine `glp_write_mps` returns zero. Otherwise, it prints an error message and returns non-zero.

3.1.3 glp_read_lp—read problem data in CPLEX LP format

Synopsis

```
int glp_read_lp(glp_prob *lp, const void *parm,
      const char *fname);
```

Description

The routine `glp_read_lp` reads problem data in CPLEX LP format from a text file. (The CPLEX LP format is described in Appendix C, page 249.)

The parameter `parm` is reserved for use in the future and must be specified as `NULL`.

The character string `fname` specifies a name of the text file to be read in. (If the file name ends with suffix '`.gz`', the file is assumed to be compressed, in which case the routine `glp_read_lp` decompresses it "on the fly".)

Note that before reading data the current content of the problem object is completely erased with the routine `glp_erase_prob`.

Returns

If the operation was successful, the routine `glp_read_lp` returns zero. Otherwise, it prints an error message and returns non-zero.

3.1.4 glp_write_lp—write problem data in CPLEX LP format

Synopsis

```
int glp_write_lp(glp_prob *lp, const void *parm,
      const char *fname);
```

Description

The routine `glp_write_lp` writes problem data in CPLEX LP format to a text file. (The CPLEX LP format is described in Appendix C, page 249.)

The parameter `parm` is reserved for use in the future and must be specified as `NULL`.

The character string `fname` specifies a name of the text file to be written out. (If the file name ends with suffix '`.gz`', the file is assumed to be compressed, in which case the routine `glp_write_lp` performs automatic compression on writing it.)

Returns

If the operation was successful, the routine `glp_write_lp` returns zero. Otherwise, it prints an error message and returns non-zero.

3.2 Routines for processing MathProg models

3.2.1 Introduction

GLPK supports the *GNU MathProg modeling language*.[1] As a rule, models written in MathProg are solved with the GLPK LP/MIP stand-alone solver `glpsol` (see Appendix D) and do not need any programming with API routines. However, for various reasons the user may need to process MathProg models directly in his/her application program, in which case he/she may use API routines described in this section. These routines provide an interface to the *MathProg translator*, a component of GLPK, which translates MathProg models into an internal code and then interprets (executes) this code.

The processing of a model written in GNU MathProg includes several steps, which should be performed in the following order:

1. *Allocating the workspace.* The translator allocates the workspace, an internal data structure used on all subsequent steps.

2. *Reading model section.* The translator reads model section and, optionally, data section from a specified text file and translates them into the internal code. If necessary, on this step data section may be ignored.

3. *Reading data section(s).* The translator reads one or more data sections from specified text file(s) and translates them into the internal code.

4. *Generating the model.* The translator executes the internal code to evaluate the content of the model objects such as sets, parameters, variables, constraints, and objectives. On this step the execution is suspended at the solve statement.

5. *Building the problem object.* The translator obtains all necessary information from the workspace and builds the standard problem object (that is, the program object of type `glp_prob`).

6. *Solving the problem.* On this step the problem object built on the previous step is passed to a solver, which solves the problem instance and stores its solution back to the problem object.

[1] The GNU MathProg modeling language is a subset of the AMPL language. For its detailed description see the document "Modeling Language GNU MathProg: Language Reference" included in the GLPK distribution.

7. *Postsolving the model.* The translator copies the solution from the problem object to the workspace and then executes the internal code from the solve statement to the end of the model. (If model has no solve statement, the translator does nothing on this step.)

8. *Freeing the workspace.* The translator frees all the memory allocated to the workspace.

Note that the MathProg translator performs no error correction, so if any of steps 2 to 7 fails (due to errors in the model), the application program should terminate processing and go to step 8.

Example 1

In this example the program reads model and data sections from input file `egypt.mod`[2] and writes the model to output file `egypt.mps` in free MPS format (see Appendix B). No solution is performed.

```
/* mplsamp1.c */

#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

int main(void)
{     glp_prob *lp;
      glp_tran *tran;
      int ret;
      lp = glp_create_prob();
      tran = glp_mpl_alloc_wksp();
      ret = glp_mpl_read_model(tran, "egypt.mod", 0);
      if (ret != 0)
      {  fprintf(stderr, "Error on translating model\n");
         goto skip;
      }
      ret = glp_mpl_generate(tran, NULL);
      if (ret != 0)
      {  fprintf(stderr, "Error on generating model\n");
         goto skip;
      }
      glp_mpl_build_prob(tran, lp);
      ret = glp_write_mps(lp, GLP_MPS_FILE, NULL, "egypt.mps");
```

[2]This is an example model included in the GLPK distribution.

```
      if (ret != 0)
         fprintf(stderr, "Error on writing MPS file\n");
skip: glp_mpl_free_wksp(tran);
      glp_delete_prob(lp);
      return 0;
}

/* eof */
```

Example 2

In this example the program reads model section from file `sudoku.mod`[3] ignoring data section in this file, reads alternative data section from file `sudoku.dat`, solves the problem instance and passes the solution found back to the model.

```
/* mplsamp2.c */

#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

int main(void)
{     glp_prob *mip;
      glp_tran *tran;
      int ret;
      mip = glp_create_prob();
      tran = glp_mpl_alloc_wksp();
      ret = glp_mpl_read_model(tran, "sudoku.mod", 1);
      if (ret != 0)
      {  fprintf(stderr, "Error on translating model\n");
         goto skip;
      }
      ret = glp_mpl_read_data(tran, "sudoku.dat");
      if (ret != 0)
      {  fprintf(stderr, "Error on translating data\n");
         goto skip;
      }
      ret = glp_mpl_generate(tran, NULL);
      if (ret != 0)
      {  fprintf(stderr, "Error on generating model\n");
         goto skip;
```

[3] This is an example model which is included in the GLPK distribution along with alternative data file `sudoku.dat`.

```
      }
      glp_mpl_build_prob(tran, mip);
      glp_simplex(mip, NULL);
      glp_intopt(mip, NULL);
      ret = glp_mpl_postsolve(tran, mip, GLP_MIP);
      if (ret != 0)
         fprintf(stderr, "Error on postsolving model\n");
skip: glp_mpl_free_wksp(tran);
      glp_delete_prob(mip);
      return 0;
}

/* eof */
```

3.2.2 glp_mpl_alloc_wksp—allocate the translator workspace

Synopsis

```
glp_tran *glp_mpl_alloc_wksp(void);
```

Description

The routine `glp_mpl_alloc_wksp` allocates the MathProg translator workspace. (Note that multiple instances of the workspace may be allocated, if necessary.)

Returns

The routine returns a pointer to the workspace, which should be used in all subsequent operations.

3.2.3 glp_mpl_read_model—read and translate model section

Synopsis

```
int glp_mpl_read_model(glp_tran *tran, const char *fname,
      int skip);
```

Description

The routine `glp_mpl_read_model` reads model section and, optionally, data section, which may follow the model section, from a text file, whose name is the character string `fname`, performs translation of model statements and data blocks, and stores all the information in the workspace.

The parameter `skip` is a flag. If the input file contains the data section and this flag is non-zero, the data section is not read as if there were no data section and a warning message is printed. This allows reading data section(s) from other file(s).

Returns

If the operation is successful, the routine returns zero. Otherwise the routine prints an error message and returns non-zero.

3.2.4 glp_mpl_read_data—read and translate data section

Synopsis

```
int glp_mpl_read_data(glp_tran *tran, const char *fname);
```

Description

The routine `glp_mpl_read_data` reads data section from a text file, whose name is the character string `fname`, performs translation of data blocks, and stores the data read in the translator workspace. If necessary, this routine may be called more than once.

Returns

If the operation is successful, the routine returns zero. Otherwise the routine prints an error message and returns non-zero.

3.2.5 glp_mpl_generate—generate the model

Synopsis

```
int glp_mpl_generate(glp_tran *tran, const char *fname);
```

Description

The routine `glp_mpl_generate` generates the model using its description stored in the translator workspace. This operation means generating all variables, constraints, and objectives, executing check and display statements, which precede the solve statement (if it is presented).

The character string `fname` specifies the name of an output text file, to which output produced by display statements should be written. If `fname` is `NULL`, the output is sent to the terminal.

Returns

If the operation is successful, the routine returns zero. Otherwise the routine prints an error message and returns non-zero.

3.2.6 glp_mpl_build_prob—build problem instance from the model

Synopsis

```
void glp_mpl_build_prob(glp_tran *tran, glp_prob *prob);
```

Description

The routine `glp_mpl_build_prob` obtains all necessary information from the translator workspace and stores it in the specified problem object `prob`. Note that before building the current content of the problem object is erased with the routine `glp_erase_prob`.

3.2.7 glp_mpl_postsolve—postsolve the model

Synopsis

```
int glp_mpl_postsolve(glp_tran *tran, glp_prob *prob,
      int sol);
```

Description

The routine `glp_mpl_postsolve` copies the solution from the specified problem object `prob` to the translator workspace and then executes all the remaining model statements, which follow the solve statement.

The parameter `sol` specifies which solution should be copied from the problem object to the workspace as follows:

`GLP_SOL` basic solution;
`GLP_IPT` interior-point solution;
`GLP_MIP` mixed integer solution.

Returns

If the operation is successful, the routine returns zero. Otherwise the routine prints an error message and returns non-zero.

3.2.8 glp_mpl_free_wksp—free the translator workspace

Synopsis

```
void glp_mpl_free_wksp(glp_tran *tran);
```

Description

The routine `glp_mpl_free_wksp` frees all the memory allocated to the translator workspace. It also frees all other resources, which are still used by the translator.

3.3 Problem solution reading/writing routines

3.3.1 glp_print_sol—write basic solution in printable format

Synopsis

```
int glp_print_sol(glp_prob *lp, const char *fname);
```

Description

The routine `glp_print_sol writes` the current basic solution of an LP problem, which is specified by the pointer `lp`, to a text file, whose name is the character string `fname`, in printable format.

Information reported by the routine `glp_print_sol` is intended mainly for visual analysis.

Returns

If no errors occurred, the routine returns zero. Otherwise the routine prints an error message and returns non-zero.

3.3.2 glp_read_sol—read basic solution from text file

Synopsis

```
int glp_read_sol(glp_prob *lp, const char *fname);
```

Description

The routine `glp_read_sol` reads basic solution from a text file whose name is specified by the parameter `fname` into the problem object.

For the file format see description of the routine `glp_write_sol`.

Returns

On success the routine returns zero, otherwise non-zero.

3.3.3 glp_write_sol—write basic solution to text file

Synopsis

```
int glp_write_sol(glp_prob *lp, const char *fname);
```

Description

The routine `glp_write_sol` writes the current basic solution to a text file whose name is specified by the parameter `fname`. This file can be read back with the routine `glp_read_sol`.

Returns

On success the routine returns zero, otherwise non-zero.

File format

The file created by the routine `glp_write_sol` is a plain text file, which contains the following information:

```
m n
p_stat d_stat obj_val
r_stat[1] r_prim[1] r_dual[1]
. . .
r_stat[m] r_prim[m] r_dual[m]
c_stat[1] c_prim[1] c_dual[1]
. . .
c_stat[n] c_prim[n] c_dual[n]
```

where:
m is the number of rows (auxiliary variables);
n is the number of columns (structural variables);
`p_stat` is the primal status of the basic solution (`GLP_UNDEF` = 1, `GLP_FEAS` = 2, `GLP_INFEAS` = 3, or `GLP_NOFEAS` = 4);
`d_stat` is the dual status of the basic solution (`GLP_UNDEF` = 1, `GLP_FEAS` = 2, `GLP_INFEAS` = 3, or `GLP_NOFEAS` = 4);
`obj_val` is the objective value;
`r_stat[i]`, $i = 1, \ldots, m$, is the status of i-th row (`GLP_BS` = 1, `GLP_NL` = 2, `GLP_NU` = 3, `GLP_NF` = 4, or `GLP_NS` = 5);
`r_prim[i]`, $i = 1, \ldots, m$, is the primal value of i-th row;
`r_dual[i]`, $i = 1, \ldots, m$, is the dual value of i-th row;

`c_stat[j]`, $j = 1, \ldots, n$, is the status of j-th column (`GLP_BS` = 1, `GLP_NL` = 2, `GLP_NU` = 3, `GLP_NF` = 4, or `GLP_NS` = 5);
`c_prim[j]`, $j = 1, \ldots, n$, is the primal value of j-th column;
`c_dual[j]`, $j = 1, \ldots, n$, is the dual value of j-th column.

3.3.4 lpx_print_sens_bnds—write bounds sensitivity information

Synopsis

```
int lpx_print_sens_bnds(glp_prob *lp, const char *fname);
```

Description

The routine `lpx_print_sens_bnds` writes the bounds for objective coefficients, right-hand-sides of constraints, and variable bounds for which the current optimal basic solution remains optimal (for LP only).

The LP is given by the pointer `lp`, and the output is written to the file specified by `fname`. The current contents of the file will be overwritten.

Information reported by the routine `lpx_print_sens_bnds` is intended mainly for visual analysis.

Returns

If no errors occurred, the routine returns zero. Otherwise the routine prints an error message and returns non-zero.

3.3.5 glp_print_ipt—write interior-point solution in printable format

Synopsis

```
int glp_print_ipt(glp_prob *lp, const char *fname);
```

Description

The routine `glp_print_ipt` writes the current interior point solution of an LP problem, which the parameter `lp` points to, to a text file, whose name is the character string `fname`, in printable format.

Information reported by the routine `glp_print_ipt` is intended mainly for visual analysis.

Returns

If no errors occurred, the routine returns zero. Otherwise the routine prints an error message and returns non-zero.

3.3.6 glp_read_ipt—read interior-point solution from text file

Synopsis

```
int glp_read_ipt(glp_prob *lp, const char *fname);
```

Description

The routine `glp_read_ipt` reads interior-point solution from a text file whose name is specified by the parameter `fname` into the problem object.

For the file format see description of the routine `glp_write_ipt`.

Returns

On success the routine returns zero, otherwise non-zero.

3.3.7 glp_write_ipt—write interior-point solution to text file

Synopsis

```
int glp_write_ipt(glp_prob *lp, const char *fname);
```

Description

The routine `glp_write_ipt` writes the current interior-point solution to a text file whose name is specified by the parameter `fname`. This file can be read back with the routine `glp_read_ipt`.

Returns

On success the routine returns zero, otherwise non-zero.

File format

The file created by the routine `glp_write_ipt` is a plain text file, which contains the following information:

```
m n
stat obj_val
r_prim[1] r_dual[1]
. . .
r_prim[m] r_dual[m]
c_prim[1] c_dual[1]
. . .
c_prim[n] c_dual[n]
```

where:
m is the number of rows (auxiliary variables);
n is the number of columns (structural variables);
`stat` is the solution status (`GLP_UNDEF` = 1 or `GLP_OPT` = 5);
`obj_val` is the objective value;
`r_prim[i]`, $i = 1, \ldots, m$, is the primal value of i-th row;
`r_dual[i]`, $i = 1, \ldots, m$, is the dual value of i-th row;
`c_prim[j]`, $j = 1, \ldots, n$, is the primal value of j-th column;
`c_dual[j]`, $j = 1, \ldots, n$, is the dual value of j-th column.

3.3.8 glp_print_mip—write MIP solution in printable format

Synopsis

```
int glp_print_mip(glp_prob *lp, const char *fname);
```

Description

The routine `glp_print_mip` writes a best known integer solution of a MIP problem, which is specified by the pointer `lp`, to a text file, whose name is the character string `fname`, in printable format.

Information reported by the routine `glp_print_mip` is intended mainly for visual analysis.

Returns

If no errors occurred, the routine returns zero. Otherwise the routine prints an error message and returns non-zero.

3.3.9 glp_read_mip—read MIP solution from text file

Synopsis

```
int glp_read_mip(glp_prob *mip, const char *fname);
```

Description

The routine `glp_read_mip` reads MIP solution from a text file whose name is specified by the parameter `fname` into the problem object.

For the file format see description of the routine `glp_write_mip`.

Returns

On success the routine returns zero, otherwise non-zero.

3.3.10 glp_write_mip—write MIP solution to text file

Synopsis

```
int glp_write_mip(glp_prob *mip, const char *fname);
```

Description

The routine `glp_write_mip` writes the current MIP solution to a text file whose name is specified by the parameter `fname`. This file can be read back with the routine `glp_read_mip`.

Returns

On success the routine returns zero, otherwise non-zero.

File format

The file created by the routine `glp_write_sol` is a plain text file, which contains the following information:

```
m n
stat obj_val
r_val[1]
. . .
r_val[m]
c_val[1]
```

```
. . .
c_val[n]
```

where:
m is the number of rows (auxiliary variables);
n is the number of columns (structural variables);
`stat` is the solution status (`GLP_UNDEF` $= 1$, `GLP_FEAS` $= 2$, `GLP_NOFEAS` $=$ 4, or `GLP_OPT` $= 5$);
`obj_val` is the objective value;
`r_val[i]`, $i = 1, \ldots, m$, is the value of i-th row;
`c_val[j]`, $j = 1, \ldots, n$, is the value of j-th column.

Chapter 4

Advanced API Routines

4.1 Background

Using vector and matrix notations LP problem (1.1)—(1.3) (see Section 1.1, page 13) can be stated as follows:

minimize (or maximize)

$$z = c^T x_S + c_0 \tag{3.1}$$

subject to linear constraints

$$x_R = A x_S \tag{3.2}$$

and bounds of variables

$$\begin{aligned} l_R &\le x_R \le u_R \\ l_S &\le x_S \le u_S \end{aligned} \tag{3.3}$$

where:
$x_R = (x_1, x_2, \ldots, x_m)$ is the vector of auxiliary variables;
$x_S = (x_{m+1}, x_{m+2}, \ldots, x_{m+n})$ is the vector of structural variables;
z is the objective function;
$c = (c_1, c_2, \ldots, c_n)$ is the vector of objective coefficients;
c_0 is the constant term ("shift") of the objective function;
$A = (a_{11}, a_{12}, \ldots, a_{mn})$ is the constraint matrix;
$l_R = (l_1, l_2, \ldots, l_m)$ is the vector of lower bounds of auxiliary variables;
$u_R = (u_1, u_2, \ldots, u_m)$ is the vector of upper bounds of auxiliary variables;
$l_S = (l_{m+1}, l_{m+2}, \ldots, l_{m+n})$ is the vector of lower bounds of structural variables;

$u_S = u_{m+1}, u_{m+2}, \ldots, u_{m+n}$ is the vector of upper bounds of structural variables.

From the simplex method's standpoint there is no difference between auxiliary and structural variables. This allows combining all these variables into one vector that leads to the following problem statement:

minimize (or maximize)

$$z = (0 \mid c)^T x + c_0 \tag{3.4}$$

subject to linear constraints

$$(I \mid -A)x = 0 \tag{3.5}$$

and bounds of variables

$$l \leq x \leq u \tag{3.6}$$

where:
$x = (x_R \mid x_S)$ is the $(m+n)$-vector of (all) variables;
$(0 \mid c)$ is the $(m+n)$-vector of objective coefficients;[1]
$(I \mid -A)$ is the *augmented* constraint $m \times (m+n)$-matrix;[2]
$l = (l_R \mid l_S)$ is the $(m+n)$-vector of lower bounds of (all) variables;
$u = (u_R \mid u_S)$ is the $(m+n)$-vector of upper bounds of (all) variables.

By definition an *LP basic solution* geometrically is a point in the space of all variables, which is the intersection of planes corresponding to active constraints[3]. The space of all variables has the dimension $m+n$, therefore, to define some basic solution we have to define $m+n$ active constraints. Note that m constraints (3.5) being linearly independent equalities are always active, so remaining n active constraints can be chosen only from bound constraints (3.6).

A variable is called *non-basic*, if its (lower or upper) bound is active, otherwise it is called *basic*. Since, as was said above, exactly n bound constraints must be active, in any basic solution there are always n non-basic variables and m basic variables. (Note that a free variable also can be

[1] Subvector 0 corresponds to objective coefficients at auxiliary variables.

[2] Note that due to auxiliary variables matrix $(I \mid -A)$ contains the unity submatrix and therefore has full rank. This means, in particular, that the system (3.5) has no linearly dependent constraints.

[3] A constraint is called *active* if in a given point it is satisfied as equality, otherwise it is called *inactive*.

non-basic. Although such variable has no bounds, we can think it as the difference between two non-negative variables, which both are non-basic in this case.)

Now consider how to determine numeric values of all variables for a given basic solution.

Let Π be an appropriate permutation matrix of the order $(m+n)$. Then we can write:

$$\begin{pmatrix} x_B \\ x_N \end{pmatrix} = \Pi \begin{pmatrix} x_R \\ x_S \end{pmatrix} = \Pi x, \tag{3.7}$$

where x_B is the vector of basic variables, x_N is the vector of non-basic variables, $x = (x_R \mid x_S)$ is the vector of all variables in the original order. In this case the system of linear constraints (3.5) can be rewritten as follows:

$$(I \mid -A)\Pi^T \Pi x = 0 \quad \Rightarrow \quad (B \mid N) \begin{pmatrix} x_B \\ x_N \end{pmatrix} = 0, \tag{3.8}$$

where

$$(B \mid N) = (I \mid -A)\Pi^T. \tag{3.9}$$

Matrix B is a square non-singular $m \times m$-matrix, which is composed from columns of the augmented constraint matrix corresponding to basic variables. It is called the *basis matrix* or simply the *basis*. Matrix N is a rectangular $m \times n$-matrix, which is composed from columns of the augmented constraint matrix corresponding to non-basic variables.

From (3.8) it follows that:

$$Bx_B + Nx_N = 0, \tag{3.10}$$

therefore,

$$x_B = -B^{-1}Nx_N. \tag{3.11}$$

Thus, the formula (3.11) shows how to determine numeric values of basic variables x_B assuming that non-basic variables x_N are fixed on their active bounds.

The $m \times n$-matrix

$$\Xi = -B^{-1}N, \tag{3.12}$$

which appears in (3.11), is called the *simplex tableau*.[4] It shows how basic variables depend on non-basic variables:

$$x_B = \Xi x_N. \tag{3.13}$$

[4]This definition corresponds to the GLPK implementation.

The system (3.13) is equivalent to the system (3.5) in the sense that they both define the same set of points in the space of (primal) variables, which satisfy to these systems. If, moreover, values of all basic variables satisfy to their bound constraints (3.3), the corresponding basic solution is called *(primal) feasible*, otherwise *(primal) infeasible*. It is understood that any (primal) feasible basic solution satisfy to all constraints (3.2) and (3.3).

The LP theory says that if LP has optimal solution, it has (at least one) basic feasible solution, which corresponds to the optimum. And the most natural way to determine whether a given basic solution is optimal or not is to use the Karush—Kuhn—Tucker optimality conditions.

For the problem statement (3.4)—(3.6) the optimality conditions are the following:[5]

$$(I \mid -A)x = 0 \tag{3.14}$$

$$(I \mid -A)^T \pi + \lambda_l + \lambda_u = \nabla z = (0 \mid c)^T \tag{3.15}$$

$$l \le x \le u \tag{3.16}$$

$$\lambda_l \ge 0, \quad \lambda_u \le 0 \quad \text{(minimization)} \tag{3.17}$$

$$\lambda_l \le 0, \quad \lambda_u \ge 0 \quad \text{(maximization)} \tag{3.18}$$

$$(\lambda_l)_k (x_k - l_k) = 0, \quad (\lambda_u)_k (x_k - u_k) = 0, \quad k = 1, 2, \ldots, m+n \tag{3.19}$$

where: $\pi = (\pi_1, \pi_2, \ldots, \pi_m)$ is a m-vector of Lagrange multipliers for equality constraints (3.5); $\lambda_l = [(\lambda_l)_1, (\lambda_l)_2, \ldots, (\lambda_l)_n]$ is a n-vector of Lagrange multipliers for lower bound constraints (3.6); $\lambda_u = [(\lambda_u)_1, (\lambda_u)_2, \ldots, (\lambda_u)_n]$ is a n-vector of Lagrange multipliers for upper bound constraints (3.6).

Condition (3.14) is the *primal* (original) system of equality constraints (3.5).

Condition (3.15) is the *dual* system of equality constraints. It requires the gradient of the objective function to be a linear combination of normals to the planes defined by constraints of the original problem.

Condition (3.16) is the primal (original) system of bound constraints (3.6).

Condition (3.17) (or (3.18) in case of maximization) is the dual system of bound constraints.

Condition (3.19) is the *complementary slackness condition*. It requires, for each original (auxiliary or structural) variable x_k, that either its (lower or upper) bound must be active, or zero bound of the corresponding Lagrange multiplier ($(\lambda_l)_k$ or $(\lambda_u)_k$) must be active.

[5] These conditions can be appiled to any solution, not only to a basic solution.

In GLPK two multipliers $(\lambda_l)_k$ and $(\lambda_u)_k$ for each primal (original) variable x_k, $k = 1, 2, \ldots, m + n$, are combined into one multiplier:

$$\lambda_k = (\lambda_l)_k + (\lambda_u)_k, \tag{3.20}$$

which is called a *dual variable* for x_k. This *cannot* lead to the ambiguity, because both lower and upper bounds of x_k cannot be active at the same time,[6] so at least one of $(\lambda_l)_k$ and $(\lambda_u)_k$ must be equal to zero, and because these multipliers have different signs, the combined multiplier, which is their sum, uniquely defines each of them.

Using dual variables λ_k the dual system of bound constraints (3.17) and (3.18) can be written in the form of so called *"rule of signs"* as follows:

Original bound constraint	Minimization			Maximization		
	$(\lambda_l)_k$	$(\lambda_u)_k$	$(\lambda_l)_k + (\lambda_u)_k$	$(\lambda_l)_k$	$(\lambda_u)_k$	$(\lambda_l)_k + (\lambda_u)_k$
$-\infty < x_k < +\infty$	$= 0$	$= 0$	$\lambda_k = 0$	$= 0$	$= 0$	$\lambda_k = 0$
$x_k \geq l_k$	≥ 0	$= 0$	$\lambda_k \geq 0$	≤ 0	$= 0$	$\lambda_k \leq 0$
$x_k \leq u_k$	$= 0$	≤ 0	$\lambda_k \leq 0$	$= 0$	≥ 0	$\lambda_k \geq 0$
$l_k \leq x_k \leq u_k$	≥ 0	≤ 0	$-\infty < \lambda_k < +\infty$	≤ 0	≥ 0	$-\infty < \lambda_k < +\infty$
$x_k = l_k = u_k$	≥ 0	≤ 0	$-\infty < \lambda_k < +\infty$	≤ 0	≥ 0	$-\infty < \lambda_k < +\infty$

May note that each primal variable x_k has its dual counterpart λ_k and vice versa. This allows applying the same partition for the vector of dual variables as (3.7):

$$\begin{pmatrix} \lambda_B \\ \lambda_N \end{pmatrix} = \Pi\lambda, \tag{3.21}$$

where λ_B is a vector of dual variables for basic variables x_B, λ_N is a vector of dual variables for non-basic variables x_N.

By definition, bounds of basic variables are inactive constraints, so in any basic solution $\lambda_B = 0$. Corresponding values of dual variables λ_N for non-basic variables x_N can be determined in the following way. From the dual system (3.15) we have:

$$(I \mid -A)^T\pi + \lambda = (0 \mid c)^T, \tag{3.22}$$

so multiplying both sides of (3.22) by matrix Π gives:

$$\Pi(I \mid -A)^T\pi + \Pi\lambda = \Pi(0 \mid c)^T. \tag{3.23}$$

[6] If x_k is a fixed variable, we can think it as double-bounded variable $l_k \leq x_k \leq u_k$, where $l_k = u_k$.

From (3.9) it follows that

$$\Pi(I \mid -A)^T = [(I \mid -A)\Pi^T]^T = (B \mid N)^T. \tag{3.24}$$

Further, we can apply the partition (3.7) also to the vector of objective coefficients (see (3.4)):

$$\begin{pmatrix} c_B \\ c_N \end{pmatrix} = \Pi \begin{pmatrix} 0 \\ c \end{pmatrix}, \tag{3.25}$$

where c_B is a vector of objective coefficients at basic variables, c_N is a vector of objective coefficients at non-basic variables. Now, substituting (3.24), (3.21), and (3.25) into (3.23), leads to:

$$(B \mid N)^T \pi + (\lambda_B \mid \lambda_N)^T = (c_B \mid c_N)^T, \tag{3.26}$$

and transposing both sides of (3.26) gives the system:

$$\begin{pmatrix} B^T \\ N^T \end{pmatrix} \pi + \begin{pmatrix} \lambda_B \\ \lambda_N \end{pmatrix} = \begin{pmatrix} c_B \\ c_T \end{pmatrix}, \tag{3.27}$$

which can be written as follows:

$$\begin{cases} B^T\pi + \lambda_B = c_B \\ N^T\pi + \lambda_N = c_N \end{cases} \tag{3.28}$$

Lagrange multipliers $\pi = (\pi_i)$ correspond to equality constraints (3.5) and therefore can have any sign. This allows resolving the first subsystem of (3.28) as follows:[7]

$$\pi = B^{-T}(c_B - \lambda_B) = -B^{-T}\lambda_B + B^{-T}c_B, \tag{3.29}$$

and substitution of π from (3.29) into the second subsystem of (3.28) gives:

$$\lambda_N = -N^T\pi + c_N = N^T B^{-T}\lambda_B + (c_N - N^T B^{-T}c_B). \tag{3.30}$$

The latter system can be written in the following final form:

$$\lambda_N = -\Xi^T\lambda_B + d, \tag{3.31}$$

where Ξ is the simplex tableau (see (3.12)), and

$$d = c_N - N^T B^{-T}c_B = c_N + \Xi^T c_B \tag{3.32}$$

is the vector of so called *reduced costs* of non-basic variables.

[7] B^{-T} means $(B^T)^{-1} = (B^{-1})^T$.

Above it was said that in any basic solution $\lambda_B = 0$, so $\lambda_N = d$ as it follows from (3.31).

The system (3.31) is equivalent to the system (3.15) in the sense that they both define the same set of points in the space of dual variables λ, which satisfy to these systems. If, moreover, values of all dual variables λ_N (i.e. reduced costs d) satisfy to their bound constraints (i.e. to the "rule of signs"; see the table above), the corresponding basic solution is called *dual feasible*, otherwise *dual infeasible*. It is understood that any dual feasible solution satisfy to all constraints (3.15) and (3.17) (or (3.18) in case of maximization).

It can be easily shown that the complementary slackness condition (3.19) is always satisfied for *any* basic solution. Therefore, a basic solution[8] is *optimal* if and only if it is primal and dual feasible, because in this case it satifies to all the optimality conditions (3.14)—(3.19).

The meaning of reduced costs $d = (d_j)$ of non-basic variables can be explained in the following way. From (3.4), (3.7), and (3.25) it follows that:

$$z = c_B^T x_B + c_N^T x_N + c_0. \tag{3.33}$$

Substituting x_B from (3.11) into (3.33) we can eliminate basic variables and express the objective only through non-basic variables:

$$\begin{aligned} z &= c_B^T(-B^{-1}Nx_N) + c_N^T x_N + c_0 = \\ &= (c_N^T - c_B^T B^{-1} N)x_N + c_0 = \\ &= (c_N - N^T B^{-T} c_B)^T x_N + c_0 = \\ &= d^T x_N + c_0. \end{aligned} \tag{3.34}$$

From (3.34) it is seen that reduced cost d_j shows how the objective function z depends on non-basic variable $(x_N)_j$ in the neighborhood of the current basic solution, i.e. while the current basis remains unchanged.

[8] It is assumed that a complete basic solution has the form (x, λ), i.e. it includes primal as well as dual variables.

4.2 LP basis routines

4.2.1 glp_bf_exists—check if the basis factorization exists

Synopsis

```
int glp_bf_exists(glp_prob *lp);
```

Returns

If the basis factorization for the current basis associated with the specified problem object exists and therefore is available for computations, the routine `glp_bf_exists` returns non-zero. Otherwise the routine returns zero.

Comments

Let the problem object have m rows and n columns. In GLPK the *basis matrix* B is a square non-singular matrix of the order m, whose columns correspond to basic (auxiliary and/or structural) variables. It is defined by the following main equality:[9]

$$(B \mid N) = (I \mid -A)\Pi^T,$$

where I is the unity matrix of the order m, whose columns correspond to auxiliary variables; A is the original constraint $m \times n$-matrix, whose columns correspond to structural variables; $(I \mid -A)$ is the augmented constraint $m \times (m + n)$-matrix, whose columns correspond to all (auxiliary and structural) variables following in the original order; Π is a permutation matrix of the order $m + n$; and N is a rectangular $m \times n$-matrix, whose columns correspond to non-basic (auxiliary and/or structural) variables.

For various reasons it may be necessary to solve linear systems with matrix B. To provide this possibility the GLPK implementation maintains an invertable form of B (that is, some representation of B^{-1}) called the *basis factorization*, which is an internal component of the problem object. Typically, the basis factorization is computed by the simplex solver, which keeps it in the problem object to be available for other computations.

Should note that any changes in the problem object, which affects the basis matrix (e.g. changing the status of a row or column, changing a basic column of the constraint matrix, removing an active constraint, etc.), invalidates the basis factorization. So before calling any API routine, which uses the basis factorization, the application program must make sure (using the

[9] For more details see Subsection 4.1, page 101.

routine `glp_bf_exists`) that the factorization exists and therefore available for computations.

4.2.2 glp_factorize—compute the basis factorization

Synopsis

```
int glp_factorize(glp_prob *lp);
```

Description

The routine `glp_factorize` computes the basis factorization for the current basis associated with the specified problem object.[10]

The basis factorization is computed from "scratch" even if it exists, so the application program may use the routine `glp_bf_exists`, and, if the basis factorization already exists, not to call the routine `glp_factorize` to prevent an extra work.

The routine `glp_factorize` *does not* compute components of the basic solution (i.e. primal and dual values).

Returns

0	The basis factorization has been successfully computed.
`GLP_EBADB`	The basis matrix is invalid, because the number of basic (auxiliary and structural) variables is not the same as the number of rows in the problem object.
`GLP_ESING`	The basis matrix is singular within the working precision.
`GLP_ECOND`	The basis matrix is ill-conditioned, i.e. its condition number is too large.

[10] The current basis is defined by the current statuses of rows (auxiliary variables) and columns (structural variables).

4.2.3 glp_bf_updated—check if the basis factorization has been updated

Synopsis

```
int glp_bf_updated(glp_prob *lp);
```

Returns

If the basis factorization has been just computed from "scratch", the routine `glp_bf_updated` returns zero. Otherwise, if the factorization has been updated at least once, the routine returns non-zero.

Comments

Updating the basis factorization means recomputing it to reflect changes in the basis matrix. For example, on every iteration of the simplex method some column of the current basis matrix is replaced by a new column that gives a new basis matrix corresponding to the adjacent basis. In this case computing the basis factorization for the adjacent basis from "scratch" (as the routine `glp_factorize` does) would be too time-consuming.

On the other hand, since the basis factorization update is a numeric computational procedure, applying it many times may lead to accumulating round-off errors. Therefore the basis is periodically refactorized (reinverted) from "scratch" (with the routine `glp_factorize`) that allows improving its numerical properties.

The routine `glp_bf_updated` allows determining if the basis factorization has been updated at least once since it was computed from "scratch".

4.2.4 glp_get_bfcp—retrieve basis factorization control parameters

Synopsis

```
void glp_get_bfcp(glp_prob *lp, glp_bfcp *parm);
```

Description

The routine `glp_get_bfcp` retrieves control parameters, which are used on computing and updating the basis factorization associated with the specified problem object.

Current values of the control parameters are stored in a `glp_bfcp` structure, which the parameter `parm` points to. For a detailed description of the structure `glp_bfcp` see comments to the routine `glp_set_bfcp` in the next subsection.

Comments

The purpose of the routine `glp_get_bfcp` is two-fold. First, it allows the application program obtaining current values of control parameters used by internal GLPK routines, which compute and update the basis factorization.

The second purpose of this routine is to provide proper values for all fields of the structure `glp_bfcp` in the case when the application program needs to change some control parameters.

4.2.5 glp_set_bfcp—change basis factorization control parameters

Synopsis

```
void glp_set_bfcp(glp_prob *lp, const glp_bfcp *parm);
```

Description

The routine `glp_set_bfcp` changes control parameters, which are used by internal GLPK routines on computing and updating the basis factorization associated with the specified problem object.

New values of the control parameters should be passed in a structure `glp_bfcp`, which the parameter `parm` points to. For a detailed description of the structure `glp_bfcp` see paragraph "Control parameters" below.

The parameter `parm` can be specified as `NULL`, in which case all control parameters are reset to their default values.

Comments

Before changing some control parameters with the routine `glp_set_bfcp` the application program should retrieve current values of all control parameters with the routine `glp_get_bfcp`. This is needed for backward compatibility, because in the future there may appear new members in the structure `glp_bfcp`.

Note that new values of control parameters come into effect on a next computation of the basis factorization, not immediately.

Example

```
glp_prob *lp;
glp_bfcp parm;
. . .
/* retrieve current values of control parameters */
glp_get_bfcp(lp, &parm);
/* change the threshold pivoting tolerance */
parm.piv_tol = 0.05;
/* set new values of control parameters */
glp_set_bfcp(lp, &parm);
. . .
```

Control parameters

This paragraph describes all basis factorization control parameters currently used in the package. Symbolic names of control parameters are names of corresponding members in the structure `glp_bfcp`.

`int type` (default: `GLP_BF_FT`)

Basis factorization type:
`GLP_BF_FT`—LU + Forrest–Tomlin update;
`GLP_BF_BG`—LU + Schur complement + Bartels–Golub update;
`GLP_BF_GR`—LU + Schur complement + Givens rotation update.
In case of `GLP_BF_FT` the update is applied to matrix U, while in cases of `GLP_BF_BG` and `GLP_BF_GR` the update is applied to the Schur complement.

`int lu_size` (default: `0`)

The initial size of the Sparse Vector Area, in non-zeros, used on computing LU-factorization of the basis matrix for the first time. If this parameter is set to 0, the initial SVA size is determined automatically.

`double piv_tol` (default: `0.10`)

Threshold pivoting (Markowitz) tolerance, $0 <$ `piv_tol` < 1, used on computing LU-factorization of the basis matrix. Element u_{ij} of the active submatrix of factor U fits to be pivot if it satisfies to the stability criterion $|u_{ij}| >=$ `piv_tol` $\cdot \max |u_{i*}|$, i.e. if it is not very small in the magnitude among other elements in the same row. Decreasing this parameter may lead to better sparsity at the expense of numerical accuracy, and vice versa.

`int piv_lim` (default: `4`)

This parameter is used on computing LU-factorization of the basis matrix and specifies how many pivot candidates needs to be considered on choosing a pivot element, `piv_lim` ≥ 1. If `piv_lim` candidates have been considered, the pivoting routine prematurely terminates the search with the best candidate found.

`int suhl` (default: `GLP_ON`)

This parameter is used on computing LU-factorization of the basis matrix. Being set to `GLP_ON` it enables applying the following heuristic proposed by Uwe Suhl: if a column of the active submatrix has no eligible pivot candidates, it is no more considered until it becomes a column singleton. In many cases this allows reducing the time needed for pivot searching. To disable this heuristic the parameter `suhl` should be set to `GLP_OFF`.

`double eps_tol` (default: `1e-15`)

Epsilon tolerance, `eps_tol` ≥ 0, used on computing LU-factorization of the basis matrix. If an element of the active submatrix of factor U is less than `eps_tol` in the magnitude, it is replaced by exact zero.

`double max_gro` (default: `1e+10`)

Maximal growth of elements of factor U, `max_gro` ≥ 1, allowable on computing LU-factorization of the basis matrix. If on some elimination step the ratio u_{big}/b_{max} (where u_{big} is the largest magnitude of elements of factor U appeared in its active submatrix during all the factorization process, b_{max} is the largest magnitude of elements of the basis matrix to be factorized), the basis matrix is considered as ill-conditioned.

`int nfs_max` (default: `100`)

Maximal number of additional row-like factors (entries of the eta file), `nfs_max` ≥ 1, which can be added to LU-factorization of the basis matrix on updating it with the Forrest–Tomlin technique. This parameter is used only once, before LU-factorization is computed for the first time, to allocate working arrays. As a rule, each update adds one new factor (however, some updates may need no addition), so this parameter limits the number of updates between refactorizations.

`double upd_tol` (default: `1e-6`)

Update tolerance, $0 <$ `upd_tol` < 1, used on updating LU-factorization of the basis matrix with the Forrest–Tomlin technique. If after updating the magnitude of some diagonal element u_{kk} of factor U becomes less than `upd_tol` $\cdot \max(|u_{k*}|, |u_{*k}|)$, the factorization is considered as inaccurate.

`int nrs_max` (default: `100`)

Maximal number of additional rows and columns, `nrs_max` ≥ 1, which can be added to LU-factorization of the basis matrix on updating it with the Schur complement technique. This parameter is used only once, before LU-factorization is computed for the first time, to allocate working arrays. As a rule, each update adds one new row and column (however, some updates may need no addition), so this parameter limits the number of updates between refactorizations.

`int rs_size` (default: `0`)

The initial size of the Sparse Vector Area, in non-zeros, used to store non-zero elements of additional rows and columns introduced on updating LU-factorization of the basis matrix with the Schur complement technique. If this parameter is set to 0, the initial SVA size is determined automatically.

4.2.6 glp_get_bhead—retrieve the basis header information

Synopsis

```
int glp_get_bhead(glp_prob *lp, int k);
```

Description

The routine `glp_get_bhead` returns the basis header information for the current basis associated with the specified problem object.

Returns

If basic variable $(x_B)_k$, $1 \le k \le m$, is i-th auxiliary variable ($1 \le i \le m$), the routine returns i. Otherwise, if $(x_B)_k$ is j-th structural variable ($1 \le j \le n$), the routine returns $m+j$. Here m is the number of rows and n is the number of columns in the problem object.

Comments

Sometimes the application program may need to know which original (auxiliary and structural) variable correspond to a given basic variable, or, that is the same, which column of the augmented constraint matrix $(I \mid -A)$ correspond to a given column of the basis matrix B.

The correspondence is defined as follows:[11]

$$\begin{pmatrix} x_B \\ x_N \end{pmatrix} = \Pi \begin{pmatrix} x_R \\ x_S \end{pmatrix} \quad \Leftrightarrow \quad \begin{pmatrix} x_R \\ x_S \end{pmatrix} = \Pi^T \begin{pmatrix} x_B \\ x_N \end{pmatrix},$$

where x_B is the vector of basic variables, x_N is the vector of non-basic variables, x_R is the vector of auxiliary variables following in their original order,[12] x_S is the vector of structural variables following in their original order, Π is a permutation matrix (which is a component of the basis factorization).

Thus, if $(x_B)_k = (x_R)_i$ is i-th auxiliary variable, the routine returns i, and if $(x_B)_k = (x_S)_j$ is j-th structural variable, the routine returns $m + j$, where m is the number of rows in the problem object.

[11] For more details see Subsection 4.1, page 101.

[12] The original order of auxiliary and structural variables is defined by the ordinal numbers of corresponding rows and columns in the problem object.

4.2.7 glp_get_row_bind—retrieve row index in the basis header

Synopsis

```
int glp_get_row_bind(glp_prob *lp, int i);
```

Returns

The routine `glp_get_row_bind` returns the index k of basic variable $(x_B)_k$, $1 \le k \le m$, which is i-th auxiliary variable (that is, the auxiliary variable corresponding to i-th row), $1 \le i \le m$, in the current basis associated with the specified problem object, where m is the number of rows. However, if i-th auxiliary variable is non-basic, the routine returns zero.

Comments

The routine `glp_get_row_bind` is an inverse to the routine `glp_get_bhead`: if `glp_get_bhead`(lp, k) returns i, `glp_get_row_bind`(lp, i) returns k, and vice versa.

4.2.8 glp_get_col_bind—retrieve column index in the basis header

Synopsis

```
int glp_get_col_bind(glp_prob *lp, int j);
```

Returns

The routine `glp_get_col_bind` returns the index k of basic variable $(x_B)_k$, $1 \le k \le m$, which is j-th structural variable (that is, the structural variable corresponding to j-th column), $1 \le j \le n$, in the current basis associated with the specified problem object, where m is the number of rows, n is the number of columns. However, if j-th structural variable is non-basic, the routine returns zero.

Comments

The routine `glp_get_col_bind` is an inverse to the routine `glp_get_bhead`: if `glp_get_bhead`(lp, k) returns $m + j$, `glp_get_col_bind`(lp, j) returns k, and vice versa.

4.2.9 glp_ftran—perform forward transformation

Synopsis

```
void glp_ftran(glp_prob *lp, double x[]);
```

Description

The routine `glp_ftran` performs forward transformation (FTRAN), i.e. it solves the system $Bx = b$, where B is the basis matrix associated with the specified problem object, x is the vector of unknowns to be computed, b is the vector of right-hand sides.

On entry to the routine elements of the vector b should be stored in locations `x[1]`, ..., `x[m]`, where m is the number of rows. On exit the routine stores elements of the vector x in the same locations.

4.2.10 glp_btran—perform backward transformation

Synopsis

```
void glp_btran(glp_prob *lp, double x[]);
```

Description

The routine `glp_btran` performs backward transformation (BTRAN), i.e. it solves the system $B^T x = b$, where B^T is a matrix transposed to the basis matrix B associated with the specified problem object, x is the vector of unknowns to be computed, b is the vector of right-hand sides.

On entry to the routine elements of the vector b should be stored in locations `x[1]`, ..., `x[m]`, where m is the number of rows. On exit the routine stores elements of the vector x in the same locations.

4.2.11 glp_warm_up—"warm up" LP basis

Synopsis

```
int glp_warm_up(glp_prob *P);
```

Description

The routine `glp_warm_up` "warms up" the LP basis for the specified problem object using current statuses assigned to rows and columns (that is, to auxiliary and structural variables).

This operation includes computing factorization of the basis matrix (if it does not exist), computing primal and dual components of basic solution, and determining the solution status.

Returns

0	The operation has been successfully performed.
`GLP_EBADB`	The basis matrix is invalid, because the number of basic (auxiliary and structural) variables is not the same as the number of rows in the problem object.
`GLP_ESING`	The basis matrix is singular within the working precision.
`GLP_ECOND`	The basis matrix is ill-conditioned, i.e. its condition number is too large.

4.3 Simplex tableau routines

4.3.1 glp_eval_tab_row—compute row of the tableau

Synopsis

```
int glp_eval_tab_row(glp_prob *lp, int k, int ind[],
      double val[]);
```

Description

The routine `glp_eval_tab_row` computes a row of the current simplex tableau (see Subsection 3.1.1, formula (3.12)), which (row) corresponds to some basic variable specified by the parameter k as follows: if $1 \le k \le m$, the basic variable is k-th auxiliary variable, and if $m + 1 \le k \le m + n$, the basic variable is $(k - m)$-th structural variable, where m is the number of rows and n is the number of columns in the specified problem object. The basis factorization must exist.

The computed row shows how the specified basic variable depends on non-basic variables:

$$x_k = (x_B)_i = \xi_{i1}(x_N)_1 + \xi_{i2}(x_N)_2 + \ldots + \xi_{in}(x_N)_n,$$

where ξ_{i1}, ξ_{i2}, ..., ξ_{in} are elements of the simplex table row, $(x_N)_1$, $(x_N)_2$, ..., $(x_N)_n$ are non-basic (auxiliary and structural) variables.

The routine stores column indices and corresponding numeric values of non-zero elements of the computed row in unordered sparse format in locations `ind[1]`, ..., `ind[len]` and `val[1]`, ..., `val[len]`, respectively, where $0 \le$ `len` $\le n$ is the number of non-zero elements in the row returned on exit.

Element indices stored in the array `ind` have the same sense as index k, i.e. indices 1 to m denote auxiliary variables while indices $m + 1$ to $m + n$ denote structural variables (all these variables are obviously non-basic by definition).

Returns

The routine `glp_eval_tab_row` returns `len`, which is the number of non-zero elements in the simplex table row stored in the arrays `ind` and `val`.

Comments

A row of the simplex table is computed as follows. At first, the routine checks that the specified variable x_k is basic and uses the permutation matrix Π

(3.7) to determine index i of basic variable $(x_B)_i$, which corresponds to x_k.

The row to be computed is i-th row of the matrix Ξ (3.12), therefore:

$$\xi_i = e_i^T \Xi = -e_i^T B^{-1} N = -(B^{-T} e_i)^T N,$$

where e_i is i-th unity vector. So the routine performs BTRAN to obtain i-th row of the inverse B^{-1}:

$$\varrho_i = B^{-T} e_i,$$

and then computes elements of the simplex table row as inner products:

$$\xi_{ij} = -\varrho_i^T N_j, \quad j = 1, 2, \ldots, n,$$

where N_j is j-th column of matrix N (3.9), which (column) corresponds to non-basic variable $(x_N)_j$. The permutation matrix Π is used again to convert indices j of non-basic columns to original ordinal numbers of auxiliary and structural variables.

4.3.2 glp_eval_tab_col—compute column of the tableau

Synopsis

```
int glp_eval_tab_col(glp_prob *lp, int k, int ind[],
      double val[]);
```

Description

The routine `glp_eval_tab_col` computes a column of the current simplex tableau (see Subsection 3.1.1, formula (3.12)), which (column) corresponds to some non-basic variable specified by the parameter k: if $1 \le k \le m$, the non-basic variable is k-th auxiliary variable, and if $m + 1 \le k \le m + n$, the non-basic variable is $(k - m)$-th structural variable, where m is the number of rows and n is the number of columns in the specified problem object. The basis factorization must exist.

The computed column shows how basic variables depends on the specified non-basic variable $x_k = (x_N)_j$:

$$\begin{aligned}
(x_B)_1 &= \ldots + \xi_{1j}(x_N)_j + \ldots \\
(x_B)_2 &= \ldots + \xi_{2j}(x_N)_j + \ldots \\
&\ldots\ldots\ldots\ldots \\
(x_B)_m &= \ldots + \xi_{mj}(x_N)_j + \ldots
\end{aligned}$$

where ξ_{1j}, ξ_{2j}, ..., ξ_{mj} are elements of the simplex table column, $(x_B)_1$, $(x_B)_2$, ..., $(x_B)_m$ are basic (auxiliary and structural) variables.

The routine stores row indices and corresponding numeric values of non-zero elements of the computed column in unordered sparse format in locations `ind[1]`, ..., `ind[len]` and `val[1]`, ..., `val[len]`, respectively, where $0 \leq$ `len` $\leq m$ is the number of non-zero elements in the column returned on exit.

Element indices stored in the array `ind` have the same sense as index k, i.e. indices 1 to m denote auxiliary variables while indices $m + 1$ to $m + n$ denote structural variables (all these variables are obviously basic by definition).

Returns

The routine `glp_eval_tab_col` returns `len`, which is the number of non-zero elements in the simplex table column stored in the arrays `ind` and `val`.

Comments

A column of the simplex table is computed as follows. At first, the routine checks that the specified variable x_k is non-basic and uses the permutation matrix Π (3.7) to determine index j of non-basic variable $(x_N)_j$, which corresponds to x_k.

The column to be computed is j-th column of the matrix Ξ (3.12), therefore:

$$\Xi_j = \Xi e_j = -B^{-1}Ne_j = -B^{-1}N_j,$$

where e_j is j-th unity vector, N_j is j-th column of matrix N (3.9). So the routine performs FTRAN to transform N_j to the simplex table column $\Xi_j = (\xi_{ij})$ and uses the permutation matrix Π to convert row indices i to original ordinal numbers of auxiliary and structural variables.

4.3.3 lpx_transform_row—transform explicitly specified row

Synopsis

```
int lpx_transform_row(glp_prob *lp, int len, int ind[],
      double val[]);
```

Description

The routine `lpx_transform_row` performs the same operation as the routine `lpx_eval_tab_row`, except that the transformed row is specified explicitly.

The explicitly specified row may be thought as a linear form:

$$x = a_1 x_{m+1} + a_2 x_{m+2} + \ldots + a_n x_{m+n}, \qquad (1)$$

where x is an auxiliary variable for this row, a_j are coefficients of the linear form, x_{m+j} are structural variables.

On entry column indices and numerical values of non-zero coefficients a_j of the transformed row should be placed in locations `ind[1]`, ..., `ind[len]` and `val[1]`, ..., `val[len]`, where `len` is number of non-zero coefficients.

This routine uses the system of equality constraints and the current basis in order to express the auxiliary variable x in (1) through the current non-basic variables (as if the transformed row were added to the problem object and the auxiliary variable x were basic), i.e. the resultant row has the form:

$$x = \alpha_1 (x_N)_1 + \alpha_2 (x_N)_2 + \ldots + \alpha_n (x_N)_n, \qquad (2)$$

where α_j are influence coefficients, $(x_N)_j$ are non-basic (auxiliary and structural) variables, n is number of columns in the specified problem object.

On exit the routine stores indices and numerical values of non-zero coefficients α_j of the resultant row (2) in locations `ind[1]`, ..., `ind[len']` and `val[1]`, ..., `val[len']`, where $0 \leq \texttt{len}' \leq n$ is the number of non-zero coefficients in the resultant row returned by the routine. Note that indices of non-basic variables stored in the array `ind` correspond to original ordinal numbers of variables: indices 1 to m mean auxiliary variables and indices $m + 1$ to $m + n$ mean structural ones.

Returns

The routine `lpx_transform_row` returns `len'`, the number of non-zero coefficients in the resultant row stored in the arrays `ind` and `val`.

4.3.4 lpx_transform_col—transform explicitly specified column

Synopsis

```
int lpx_transform_col(glp_prob *lp, int len, int ind[],
      double val[]);
```

Description

The routine `lpx_transform_col` performs the same operation as the routine `lpx_eval_tab_col`, except that the transformed column is specified explicitly.

The explicitly specified column may be thought as it were added to the original system of equality constraints:

$$\begin{aligned} x_1 &= a_{11}x_{m+1} + \ldots + a_{1n}x_{m+n} + a_1x \\ x_2 &= a_{21}x_{m+1} + \ldots + a_{2n}x_{m+n} + a_2x \\ &\ldots\ldots\ldots \\ x_m &= a_{m1}x_{m+1} + \ldots + a_{mn}x_{m+n} + a_mx \end{aligned} \tag{1}$$

where x_i are auxiliary variables, x_{m+j} are structural variables (presented in the problem object), x is a structural variable for the explicitly specified column, a_i are constraint coefficients for x.

On entry row indices and numerical values of non-zero coefficients a_i of the transformed column should be placed in locations `ind[1]`, ..., `ind[len]` and `val[1]`, ..., `val[len]`, where `len` is number of non-zero coefficients.

This routine uses the system of equality constraints and the current basis in order to express the current basic variables through the structural variable x in (1) (as if the transformed column were added to the problem object and the variable x were non-basic):

$$\begin{aligned} (x_B)_1 &= \ldots + \alpha_1x \\ (x_B)_2 &= \ldots + \alpha_2x \\ &\ldots\ldots\ldots \\ (x_B)_m &= \ldots + \alpha_mx \end{aligned} \tag{2}$$

where α_i are influence coefficients, x_B are basic (auxiliary and structural) variables, m is number of rows in the specified problem object.

On exit the routine stores indices and numerical values of non-zero coefficients α_i of the resultant column (2) in locations `ind[1]`, ..., `ind[len']` and `val[1]`, ..., `val[len']`, where $0 \leq \texttt{len}' \leq m$ is the number of non-zero

coefficients in the resultant column returned by the routine. Note that indices of basic variables stored in the array `ind` correspond to original ordinal numbers of variables, i.e. indices 1 to m mean auxiliary variables, indices $m + 1$ to $m + n$ mean structural ones.

Returns

The routine `lpx_transform_col` returns `len'`, the number of non-zero coefficients in the resultant column stored in the arrays `ind` and `val`.

4.3.5 lpx_prim_ratio_test—perform primal ratio test

Synopsis

```
int lpx_prim_ratio_test(glp_prob *lp, int len, int ind[],
      double val[], int how, double tol);
```

Description

The routine `lpx_prim_ratio_test` performs the primal ratio test for an explicitly specified column of the simplex table.

The primal basic solution associated with an LP problem object, which the parameter `lp` points to, should be feasible. No components of the LP problem object are changed by the routine.

The explicitly specified column of the simplex table shows how the basic variables x_B depend on some non-basic variable y (which is not necessarily presented in the problem object):

$$\begin{aligned} (x_B)_1 &= \ldots + \alpha_1 y \\ (x_B)_2 &= \ldots + \alpha_2 y \\ &\cdots\cdots\cdots \\ (x_B)_m &= \ldots + \alpha_m y \end{aligned} \tag{1}$$

The column (1) is specifed on entry to the routine using the sparse format. Ordinal numbers of basic variables $(x_B)_i$ should be placed in locations `ind[1]`, ..., `ind[len]`, where ordinal number 1 to m denote auxiliary variables, and ordinal numbers $m+1$ to $m+n$ denote structural variables. The corresponding non-zero coefficients α_i should be placed in locations `val[1]`, ..., `val[len]`. The arrays `ind` and `val` are not changed by the routine.

The parameter `how` specifies in which direction the variable y changes on entering the basis: $+1$ means increasing, -1 means decreasing.

The parameter `tol` is a relative tolerance (small positive number) used by the routine to skip small α_i in the column (1).

The routine determines the ordinal number of a basic variable (among specified in `ind[1]`, ..., `ind[len]`), which reaches its (lower or upper) bound first before any other basic variables do and which therefore should leave the basis instead the variable y in order to keep primal feasibility, and returns it on exit. If the choice cannot be made (i.e. if the adjacent basic solution is primal unbounded due to y), the routine returns zero.

Note

If the non-basic variable y is presented in the LP problem object, the column (1) can be computed using the routine `lpx_eval_tab_col`. Otherwise it can be computed using the routine `lpx_transform_col`.

Returns

The routine `lpx_prim_ratio_test` returns the ordinal number of some basic variable $(x_B)_i$, which should leave the basis instead the variable y in order to keep primal feasibility. If the adjacent basic solution is primal unbounded and therefore the choice cannot be made, the routine returns zero.

4.3.6 lpx_dual_ratio_test—perform dual ratio test

Synopsis

```
int lpx_dual_ratio_test(glp_prob *lp, int len, int ind[],
      double val[], int how, double tol);
```

Description

The routine `lpx_dual_ratio_test` performs the dual ratio test for an explicitly specified row of the simplex table.

The dual basic solution associated with an LP problem object, which the parameter `lp` points to, should be feasible. No components of the LP problem object are changed by the routine.

The explicitly specified row of the simplex table is a linear form, which shows how some basic variable y (not necessarily presented in the problem object) depends on non-basic variables x_N:

$$y = \alpha_1(x_N)_1 + \alpha_2(x_N)_2 + \ldots + \alpha_n(x_N)_n. \qquad (1)$$

The linear form (1) is specified on entry to the routine using the sparse format. Ordinal numbers of non-basic variables $(x_N)_j$ should be placed in locations `ind[1]`, ..., `ind[len]`, where ordinal numbers 1 to m denote auxiliary variables, and ordinal numbers $m + 1$ to $m + n$ denote structural variables. The corresponding non-zero coefficients α_j should be placed in locations `val[1]`, ..., `val[len]`. The arrays `ind` and `val` are not changed by the routine.

The parameter `how` specifies in which direction the variable y changes on leaving the basis: $+1$ means increasing, -1 means decreasing.

The parameter `tol` is a relative tolerance (small positive number) used by the routine to skip small α_j in the form (1).

The routine determines the ordinal number of some non-basic variable (among specified in `ind[1]`, ..., `ind[len]`), whose reduced cost reaches its (zero) bound first before this happens for any other non-basic variables and which therefore should enter the basis instead the variable y in order to keep dual feasibility, and returns it on exit. If the choice cannot be made (i.e. if the adjacent basic solution is dual unbounded due to y), the routine returns zero.

Note

If the basic variable y is presented in the LP problem object, the row (1) can be computed using the routine `lpx_eval_tab_row`. Otherwise it can be computed using the routine `lpx_transform_row`.

Returns

The routine `lpx_dual_ratio_test` returns the ordinal number of some non-basic variable $(x_N)_j$, which should enter the basis instead the variable y in order to keep dual feasibility. If the adjacent basic solution is dual unbounded and therefore the choice cannot be made, the routine returns zero.

Chapter 5

Branch-and-Cut API Routines

5.1 Introduction

5.1.1 Using the callback routine

The GLPK MIP solver based on the branch-and-cut method allows the application program to control the solution process. This is attained by means of the user-defined callback routine, which is called by the solver at various points of the branch-and-cut algorithm.

The callback routine passed to the MIP solver should be written by the user and has the following specification:[1]

```
void foo_bar(glp_tree *tree, void *info);
```

where `tree` is a pointer to the data structure `glp_tree`, which should be used on subsequent calls to branch-and-cut interface routines, and `info` is a transit pointer passed to the routine `glp_intopt`, which may be used by the application program to pass some external data to the callback routine.

The callback routine is passed to the MIP solver through the control parameter structure `glp_iocp` (see Chapter "Basic API Routines", Section "Mixed integer programming routines", Subsection "Solve MIP problem with the branch-and-cut method") as follows:

[1] The name `foo_bar` used here is a placeholder for the callback routine name.

```
   glp_prob *mip;
   glp_iocp parm;
   . . .
   glp_init_iocp(&parm);
   . . .
   parm.cb_func = foo_bar;
   parm.cb_info = ... ;
   ret = glp_intopt(mip, &parm);
   . . .
```

To determine why it is being called by the MIP solver the callback routine should use the routine `glp_ios_reason` (described in this section below), which returns a code indicating the reason for calling. Depending on the reason the callback routine may perform necessary actions to control the solution process.

The reason codes, which correspond to various point of the branch-and-cut algorithm implemented in the MIP solver, are described in Subsection "Reasons for calling the callback routine" below.

To ignore calls for reasons, which are not processed by the callback routine, it should just return to the MIP solver doing nothing. For example:

```
void foo_bar(glp_tree *tree, void *info)
{     . . .
      switch (glp_ios_reason(tree))
      {  case GLP_IBRANCH:
            . . .
            break;
         case GLP_ISELECT:
            . . .
            break;
         default:
            /* ignore call for other reasons */
            break;
      }
      return;
}
```

To control the solution process as well as to obtain necessary information the callback routine may use the branch-and-cut API routines described in this chapter. Names of all these routines begin with '`glp_ios_`'.

5.1.2 Branch-and-cut algorithm

This section gives a schematic description of the branch-and-cut algorithm as it is implemented in the GLPK MIP solver.

1. Initialization

Set $L := \{P_0\}$, where L is the *active list* (i.e. the list of active subproblems), P_0 is the original MIP problem to be solved.

Set $\overline{z} := +\infty$ (in case of minimization) or $\overline{z} := -\infty$ (in case of maximization), where $\overline{z}$ is *incumbent value*, i.e. an upper (minimization) or lower (maximization) global bound for z^*, the optimal objective value for P^0.

2. Subproblem selection

If $L = \emptyset$ then GO TO 9.

Select $P \in L$, i.e. make active subproblem P current.

3. Solving LP relaxation

Solve P_{LP}, which is LP relaxation of P.

If P_{LP} has no primal feasible solution then GO TO 8.

Let z^*_{LP} be the optimal objective value for P_{LP}.

If $z^*_{LP} \geq \overline{z}$ (in case of minimization) or $z^*_{LP} \leq \overline{z}$ (in case of maximization) then GO TO 8.

4. Adding "lazy" constraints

Let x^*_{LP} be the optimal solution to P_{LP}.

If there are "lazy" constraints (i.e. essential constraints not included in the original MIP problem P_0), which are violated at the optimal point x^*_{LP}, add them to P, and GO TO 3.

5. Check for integrality

Let x_j be a variable, which is required to be integer, and let $x^*_j \in x^*_{LP}$ be its value in the optimal solution to P_{LP}.

If x^*_j is integral for all integer variables, then a better integer feasible solution is found. Store its components, set $\overline{z} := z^*_{LP}$, and GO TO 8.

6. Adding cutting planes

If there are cutting planes (i.e. valid constraints for P), which are violated at the optimal point x^*_{LP}, add them to P, and GO TO 3.

7. Branching

Select *branching variable* x_j, i.e. a variable, which is required to be integer, and whose value $x^*_j \in x^*_{LP}$ is fractional in the optimal solution to P_{LP}.

Create new subproblem P_D (so called *down branch*), which is identical to the current subproblem P with exception that the upper bound of x_j is replaced by $\lfloor x_j^* \rfloor$. (For example, if $x_j^* = 3.14$, the new upper bound of x_j in the down branch will be $\lfloor 3.14 \rfloor = 3$.)

Create new subproblem P_U (so called *up branch*), which is identical to the current subproblem P with exception that the lower bound of x_j is replaced by $\lceil x_j^* \rceil$. (For example, if $x_j^* = 3.14$, the new lower bound of x_j in the up branch will be $\lceil 3.14 \rceil = 4$.)

Set $L := L \backslash \{P\} \cup \{P_D, P_U\}$, i.e. remove the current subproblem P from the active list and add two new subproblems P_D and P_U to the active list. Then GO TO 2.

8. Pruning

Remove from the active list L all subproblems (including the current one), whose local bound $\widetilde{z}$ is not better than the global bound $\overline{z}$, i.e. set $L := L \backslash \{P\}$ for all P, where $\widetilde{z} \geq \overline{z}$ (in case of minimization) or $\widetilde{z} \leq \overline{z}$ (in case of maximization), and then GO TO 2.

The local bound $\widetilde{z}$ for subproblem P is an lower (minimization) or upper (maximization) bound for integer optimal solution to *this* subproblem (not to the original problem). This bound is local in the sense that only subproblems in the subtree rooted at node P cannot have better integer feasible solutions. Note that the local bound is not necessarily the optimal objective value to LP relaxation P_{LP}.

9. Termination

If $\overline{z} = +\infty$ (in case of minimization) or $\overline{z} = -\infty$ (in case of maximization), the original problem P_0 has no integer feasible solution. Otherwise, the last integer feasible solution stored on step 5 is the integer optimal solution to the original problem P_0. STOP.

5.1.3 The search tree

On the branching step of the branch-and-cut algorithm the current subproblem is divided into two[2] new subproblems, so the set of all subproblems can be represented in the form of a rooted tree, which is called the *search* or *branch-and-bound* tree. An example of the search tree is shown on Fig. 1. Each node of the search tree corresponds to a subproblem, so the terms 'node' and 'subproblem' may be used synonymously.

[2] In more general cases the current subproblem may be divided into more than two subproblems. However, currently such feature is not used in GLPK.

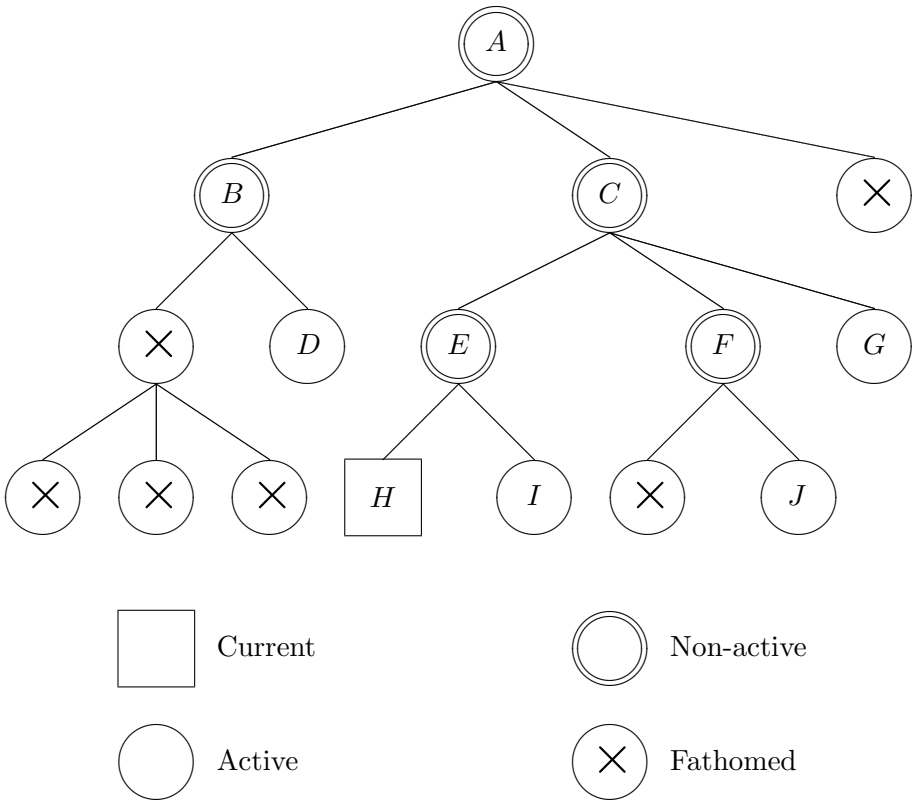


Fig. 1. An example of the search tree

In GLPK each node may have one of the following four statuses:

- *current node* is the active node currently being processed;
- *active node* is a leaf node, which still has to be processed;
- *non-active node* is a node, which has been processed, but not fathomed;
- *fathomed node* is a node, which has been processed and fathomed.

In the data structure representing the search tree GLPK keeps only current, active, and non-active nodes. Once a node has been fathomed, it is removed from the tree data structure.

Being created each node of the search tree is assigned a distinct positive integer called the *subproblem reference number*, which may be used by the application program to specify a particular node of the tree. The root node corresponding to the original problem to be solved is always assigned the reference number 1.

5.1.4 Current subproblem

The current subproblem is a MIP problem corresponding to the current node of the search tree. It is represented as the GLPK problem object (`glp_prob`) that allows the application program using API routines to access its content in the standard way. If the MIP presolver is not used, it is the original problem object passed to the routine `glp_intopt`; otherwise, it is an internal problem object built by the MIP presolver.

Note that the problem object is used by the MIP solver itself during the solution process for various purposes (to solve LP relaxations, to perfom branching, etc.), and even if the MIP presolver is not used, the current content of the problem object may differ from its original content. For example, it may have additional rows, bounds of some rows and columns may be changed, etc. In particular, LP segment of the problem object corresponds to LP relaxation of the current subproblem. However, on exit from the MIP solver the content of the problem object is restored to its original state.

To obtain information from the problem object the application program may use any API routines, which do not change the object. Using API routines, which change the problem object, is restricted to stipulated cases.

5.1.5 The cut pool

The *cut pool* is a set of cutting plane constraints maintained by the MIP solver. It is used by the GLPK cut generation routines and may be used by the application program in the same way, i.e. rather than to add cutting plane constraints directly to the problem object the application program may store them to the cut pool. In the latter case the solver looks through the cut pool, selects efficient constraints, and adds them to the problem object.

5.1.6 Reasons for calling the callback routine

The callback routine may be called by the MIP solver for the following reasons.

Request for subproblem selection

The callback routine is called with the reason code `GLP_ISELECT` if the current subproblem has been fathomed and therefore there is no current subproblem.

In response the callback routine may select some subproblem from the active list and pass its reference number to the solver using the routine `glp_ios_select_node`, in which case the solver continues the search from the specified active subproblem. If no selection is made by the callback routine, the solver uses a backtracking technique specified by the control parameter `bt_tech`.

To explore the active list (i.e. active nodes of the branch-and-bound tree) the callback routine may use the routines `glp_ios_next_node` and `glp_ios_prev_node`.

Request for preprocessing

The callback routine is called with the reason code `GLP_IPREPRO` if the current subproblem has just been selected from the active list and its LP relaxation is not solved yet.

In response the callback routine may perform some preprocessing of the current subproblem like tightening bounds of some variables or removing bounds of some redundant constraints.

Request for row generation

The callback routine is called with the reason code `GLP_IROWGEN` if LP relaxation of the current subproblem has just been solved to optimality and its objective value is better than the best known integer feasible solution.

In response the callback routine may add one or more "lazy" constraints (rows), which are violated by the current optimal solution of LP relaxation, using API routines `glp_add_rows`, `glp_set_row_name`, `glp_set_row_bnds`, and `glp_set_mat_row`, in which case the solver will perform re-optimization of LP relaxation. If there are no violated constraints, the callback routine should just return.

Optimal solution components for LP relaxation can be obtained with API routines `glp_get_obj_val`, `glp_get_row_prim`, `glp_get_row_dual`, `glp_get_col_prim`, and `glp_get_col_dual`.

Request for heuristic solution

The callback routine is called with the reason code `GLP_IHEUR` if LP relaxation of the current subproblem being solved to optimality is integer infeasible (i.e. values of some structural variables of integer kind are fractional), though its objective value is better than the best known integer feasible solution.

In response the callback routine may try applying a primal heuristic to find an integer feasible solution,[3] which is better than the best known one. In case of success the callback routine may store such better solution in the problem object using the routine `glp_ios_heur_sol`.

Request for cut generation

The callback routine is called with the reason code `GLP_ICUTGEN` if LP relaxation of the current subproblem being solved to optimality is integer infeasible (i.e. values of some structural variables of integer kind are fractional), though its objective value is better than the best known integer feasible solution.

In response the callback routine may reformulate the *current* subproblem (before it will be splitted up due to branching) by adding to the problem object one or more *cutting plane constraints*, which cut off the fractional optimal point from the MIP polytope.[4]

Adding cutting plane constraints may be performed in two ways. One way is the same as for the reason code `GLP_IROWGEN` (see above), in which case the callback routine adds new rows corresponding to cutting plane constraints directly to the current subproblem.

The other way is to add cutting plane constraints to the *cut pool*, a set of cutting plane constraints maintained by the solver, rather than directly to the current subproblem. In this case after return from the callback routine the solver looks through the cut pool, selects efficient cutting plane constraints, adds them to the current subproblem, drops other constraints, and then performs re-optimization.

Request for branching

The callback routine is called with the reason code `GLP_IBRANCH` if LP relaxation of the current subproblem being solved to optimality is integer infeasible (i.e. values of some structural variables of integer kind are fractional), though its objective value is better than the best known integer feasible solution.

In response the callback routine may choose some variable suitable for branching (i.e. integer variable, whose value in optimal solution to LP relaxation of the current subproblem is fractional) and pass its ordinal number

[3] Integer feasible to the original MIP problem, not to the current subproblem.

[4] Since these constraints are added to the current subproblem, they may be globally as well as locally valid.

to the solver using the routine `glp_ios_branch_upon`, in which case the solver splits the current subproblem in two new subproblems and continues the search. If no choice is made by the callback routine, the solver uses a branching technique specified by the control parameter `br_tech`.

Better integer solution found

The callback routine is called with the reason code `GLP_IBINGO` if LP relaxation of the current subproblem being solved to optimality is integer feasible (i.e. values of all structural variables of integer kind are integral within the working precision) and its objective value is better than the best known integer feasible solution.

Optimal solution components for LP relaxation can be obtained in the same way as for the reason code `GLP_IROWGEN` (see above).

Components of the new MIP solution can be obtained with API routines `glp_mip_obj_val`, `glp_mip_row_val`, and `glp_mip_col_val`. Note, however, that due to row/cut generation there may be additional rows in the problem object.

The difference between optimal solution to LP relaxation and corresponding MIP solution is that in the former case some structural variables of integer kind (namely, basic variables) may have values, which are close to nearest integers within the working precision, while in the latter case all such variables have exact integral values.

The reason `GLP_IBINGO` is intended only for informational purposes, so the callback routine should not modify the problem object in this case.

5.2 Basic routines

5.2.1 glp_ios_reason—determine reason for calling the callback routine

Synopsis

```
int glp_ios_reason(glp_tree *tree);
```

Returns

The routine `glp_ios_reason` returns a code, which indicates why the user-defined callback routine is being called:

`GLP_ISELECT`—request for subproblem selection;
`GLP_IPREPRO`—request for preprocessing;
`GLP_IROWGEN`—request for row generation;
`GLP_IHEUR` —request for heuristic solution;
`GLP_ICUTGEN`—request for cut generation;
`GLP_IBRANCH`—request for branching;
`GLP_IBINGO` —better integer solution found.

5.2.2 glp_ios_get_prob—access the problem object

Synopsis

```
glp_prob *glp_ios_get_prob(glp_tree *tree);
```

Description

The routine `glp_ios_get_prob` can be called from the user-defined callback routine to access the problem object, which is used by the MIP solver. It is the original problem object passed to the routine `glp_intopt` if the MIP presolver is not used; otherwise it is an internal problem object built by the presolver.

Returns

The routine `glp_ios_get_prob` returns a pointer to the problem object used by the MIP solver.

Comments

To obtain various information about the problem instance the callback routine can access the problem object (i.e. the object of type `glp_prob`) using the routine `glp_ios_get_prob`. It is the original problem object passed to the routine `glp_intopt` if the MIP presolver is not used; otherwise it is an internal problem object built by the presolver.

5.2.3 glp_ios_row_attr—determine additional row attributes

Synopsis

```
void glp_ios_row_attr(glp_tree *tree, int i, glp_attr *attr);
```

Description

The routine `glp_ios_row_attr` retrieves additional attributes of i-th row of the current subproblem and stores them in the structure `glp_attr`, which the parameter `attr` points to.

The structure `glp_attr` has the following fields:

`int level`
: Subproblem level at which the row was created. (If `level` = 0, the row was added either to the original problem object passed to the routine `glp_intopt` or to the root subproblem on generating "lazy" or/and cutting plane constraints.)

`int origin`
: The row origin flag:
`GLP_RF_REG` —regular constraint;
`GLP_RF_LAZY`—"lazy" constraint;
`GLP_RF_CUT` —cutting plane constraint.

`int klass`
: The row class descriptor, which is a number passed to the routine `glp_ios_add_row` as its third parameter. If the row is a cutting plane constraint generated by the solver, its class may be the following:
`GLP_RF_GMI` —Gomory's mixed integer cut;
`GLP_RF_MIR` —mixed integer rounding cut;
`GLP_RF_COV` —mixed cover cut;
`GLP_RF_CLQ` —clique cut.

5.2.4 glp_ios_mip_gap—compute relative MIP gap

Synopsis

```
double glp_ios_mip_gap(glp_tree *tree);
```

Description

The routine `glp_ios_mip_gap` computes the relative MIP gap (also called *duality gap*) with the following formula:

$$\texttt{gap} = \frac{|\texttt{best\_mip} - \texttt{best\_bnd}|}{|\texttt{best\_mip}| + \texttt{DBL\_EPSILON}}$$

where `best_mip` is the best integer feasible solution found so far, `best_bnd` is the best (global) bound. If no integer feasible solution has been found yet, `gap` is set to `DBL_MAX`.

Returns

The routine `glp_ios_mip_gap` returns the relative MIP gap.

Comments

The relative MIP gap is used to measure the quality of the best integer feasible solution found so far, because the optimal solution value z^* for the original MIP problem always lies in the range

$$\texttt{best\_bnd} \leq z^* \leq \texttt{best\_mip}$$

in case of minimization, or in the range

$$\texttt{best\_mip} \leq z^* \leq \texttt{best\_bnd}$$

in case of maximization.

To express the relative MIP gap in percents the value returned by the routine `glp_ios_mip_gap` should be multiplied by 100%.

5.2.5 glp_ios_node_data—access application-specific data

Synopsis

```
void *glp_ios_node_data(glp_tree *tree, int p);
```

Description

The routine `glp_ios_node_data` allows the application accessing a memory block allocated for the subproblem (which may be active or inactive), whose reference number is p.

The size of the block is defined by the control parameter `cb_size` passed to the routine `glp_intopt`. The block is initialized by binary zeros on creating corresponding subproblem, and its contents is kept until the subproblem will be removed from the tree.

The application may use these memory blocks to store specific data for each subproblem.

Returns

The routine `glp_ios_node_data` returns a pointer to the memory block for the specified subproblem. Note that if `cb_size` = 0, the routine returns a null pointer.

5.2.6 glp_ios_select_node—select subproblem to continue the search

Synopsis

```
void glp_ios_select_node(glp_tree *tree, int p);
```

Description

The routine `glp_ios_select_node` can be called from the user-defined callback routine in response to the reason `GLP_ISELECT` to select an active subproblem, whose reference number is p. The search will be continued from the subproblem selected.

5.2.7 glp_ios_heur_sol—provide solution found by heuristic

Synopsis

```
int glp_ios_heur_sol(glp_tree *tree, const double x[]);
```

Description

The routine `glp_ios_heur_sol` can be called from the user-defined callback routine in response to the reason `GLP_IHEUR` to provide an integer feasible solution found by a primal heuristic.

Primal values of *all* variables (columns) found by the heuristic should be placed in locations $x[1], \ldots, x[n]$, where n is the number of columns in the original problem object. Note that the routine `glp_ios_heur_sol` does *not* check primal feasibility of the solution provided.

Using the solution passed in the array x the routine computes value of the objective function. If the objective value is better than the best known integer feasible solution, the routine computes values of auxiliary variables (rows) and stores all solution components in the problem object.

Returns

If the provided solution is accepted, the routine `glp_ios_heur_sol` returns zero. Otherwise, if the provided solution is rejected, the routine returns non-zero.

5.2.8 glp_ios_can_branch—check if can branch upon specified variable

Synopsis

```
int glp_ios_can_branch(glp_tree *tree, int j);
```

Returns

If j-th variable (column) can be used to branch upon, the routine returns non-zero, otherwise zero.

5.2.9 glp_ios_branch_upon—choose variable to branch upon

Synopsis

```
void glp_ios_branch_upon(glp_tree *tree, int j, int sel);
```

Description

The routine `glp_ios_branch_upon` can be called from the user-defined callback routine in response to the reason `GLP_IBRANCH` to choose a branching variable, whose ordinal number is j. Should note that only variables, for which the routine `glp_ios_can_branch` returns non-zero, can be used to branch upon.

The parameter `sel` is a flag that indicates which branch (subproblem) should be selected next to continue the search:

`GLP_DN_BRNCH`—select down-branch;

`GLP_UP_BRNCH`—select up-branch;

`GLP_NO_BRNCH`—use general selection technique.

Comments

On branching the solver removes the current active subproblem from the active list and creates two new subproblems (*down-* and *up-branches*), which are added to the end of the active list. Note that the down-branch is created before the up-branch, so the last active subproblem will be the up-branch.

The down- and up-branches are identical to the current subproblem with exception that in the down-branch the upper bound of x_j, the variable chosen to branch upon, is replaced by $\lfloor x_j^* \rfloor$, while in the up-branch the lower bound of x_j is replaced by $\lceil x_j^* \rceil$, where x_j^* is the value of x_j in optimal solution to LP relaxation of the current subproblem. For example, if $x_j^* = 3.14$, the new upper bound of x_j in the down-branch is $\lfloor 3.14 \rfloor = 3$, and the new lower bound in the up-branch is $\lceil 3.14 \rceil = 4$.)

Additionally the callback routine may select either down- or up-branch, from which the solver will continue the search. If none of the branches is selected, a general selection technique will be used.

5.2.10 glp_ios_terminate—terminate the solution process

Synopsis

```
void glp_ios_terminate(glp_tree *tree);
```

Description

The routine `glp_ios_terminate` sets a flag indicating that the MIP solver should prematurely terminate the search.

5.3 The search tree exploring routines

5.3.1 glp_ios_tree_size—determine size of the search tree

Synopsis

```
void glp_ios_tree_size(glp_tree *tree, int *a_cnt, int *n_cnt,
      int *t_cnt);
```

Description

The routine `glp_ios_tree_size` stores the following three counts which characterize the current size of the search tree:

`a_cnt` is the current number of active nodes, i.e. the current size of the active list;

`n_cnt` is the current number of all (active and inactive) nodes;

`t_cnt` is the total number of nodes including those which have been already removed from the tree. This count is increased whenever a new node appears in the tree and never decreased.

If some of the parameters `a_cnt`, `n_cnt`, `t_cnt` is a null pointer, the corresponding count is not stored.

5.3.2 glp_ios_curr_node—determine current active subproblem

Synopsis

```
int glp_ios_curr_node(glp_tree *tree);
```

Returns

The routine `glp_ios_curr_node` returns the reference number of the current active subproblem. However, if the current subproblem does not exist, the routine returns zero.

5.3.3 glp_ios_next_node—determine next active subproblem

Synopsis

```
int glp_ios_next_node(glp_tree *tree, int p);
```

Returns

If the parameter p is zero, the routine `glp_ios_next_node` returns the reference number of the first active subproblem. However, if the tree is empty, zero is returned.

If the parameter p is not zero, it must specify the reference number of some active subproblem, in which case the routine returns the reference number of the next active subproblem. However, if there is no next active subproblem in the list, zero is returned.

All subproblems in the active list are ordered chronologically, i.e. subproblem A precedes subproblem B if A was created before B.

5.3.4 glp_ios_prev_node—determine previous active subproblem

Synopsis

```
int glp_ios_prev_node(glp_tree *tree, int p);
```

Returns

If the parameter p is zero, the routine `glp_ios_prev_node` returns the reference number of the last active subproblem. However, if the tree is empty, zero is returned.

If the parameter p is not zero, it must specify the reference number of some active subproblem, in which case the routine returns the reference number of the previous active subproblem. However, if there is no previous active subproblem in the list, zero is returned.

All subproblems in the active list are ordered chronologically, i.e. subproblem A precedes subproblem B if A was created before B.

5.3.5 glp_ios_up_node—determine parent subproblem

Synopsis

```
int glp_ios_up_node(glp_tree *tree, int p);
```

Returns

The parameter p must specify the reference number of some (active or inactive) subproblem, in which case the routine `iet_get_up_node` returns the reference number of its parent subproblem. However, if the specified subproblem is the root of the tree and, therefore, has no parent, the routine returns zero.

5.3.6 glp_ios_node_level—determine subproblem level

Synopsis

```
int glp_ios_node_level(glp_tree *tree, int p);
```

Returns

The routine `glp_ios_node_level` returns the level of the subproblem, whose reference number is p, in the branch-and-bound tree. (The root subproblem has level 0, and the level of any other subproblem is the level of its parent plus one.)

5.3.7 glp_ios_node_bound—determine subproblem local bound

Synopsis

```
double glp_ios_node_bound(glp_tree *tree, int p);
```

Returns

The routine `glp_ios_node_bound` returns the local bound for (active or inactive) subproblem, whose reference number is p.

Comments

The local bound for subproblem p is an lower (minimization) or upper (maximization) bound for integer optimal solution to *this* subproblem (not to the

original problem). This bound is local in the sense that only subproblems in the subtree rooted at node p cannot have better integer feasible solutions.

On creating a subproblem (due to the branching step) its local bound is inherited from its parent and then may get only stronger (never weaker). For the root subproblem its local bound is initially set to `-DBL_MAX` (minimization) or `+DBL_MAX` (maximization) and then improved as the root LP relaxation has been solved.

Note that the local bound is not necessarily the optimal objective value to corresponding LP relaxation.

5.3.8 glp_ios_best_node—find active subproblem with best local bound

Synopsis

```
int glp_ios_best_node(glp_tree *tree);
```

Returns

The routine `glp_ios_best_node` returns the reference number of the active subproblem, whose local bound is best (i.e. smallest in case of minimization or largest in case of maximization). However, if the tree is empty, the routine returns zero.

Comments

The best local bound is an lower (minimization) or upper (maximization) bound for integer optimal solution to the original MIP problem.

5.4 The cut pool routines

5.4.1 glp_ios_pool_size—determine current size of the cut pool

Synopsis

```
int glp_ios_pool_size(glp_tree *tree);
```

Returns

The routine `glp_ios_pool_size` returns the current size of the cut pool, that is, the number of cutting plane constraints currently added to it.

5.4.2 glp_ios_add_row—add constraint to the cut pool

Synopsis

```
int glp_ios_add_row(glp_tree *tree, const char *name,
      int klass, int flags, int len, const int ind[],
      const double val[], int type, double rhs);
```

Description

The routine `glp_ios_add_row` adds specified row (cutting plane constraint) to the cut pool.

The cutting plane constraint should have the following format:

$$\sum_{j \in J} a_j x_j \left\{ \begin{matrix} \geq \\ \leq \end{matrix} \right\} b,$$

where J is a set of indices (ordinal numbers) of structural variables, a_j are constraint coefficients, x_j are structural variables, b is the right-hand side.

The parameter `name` specifies a symbolic name assigned to the constraint (1 up to 255 characters). If it is `NULL` or an empty string, no name is assigned.

The parameter `klass` specifies the constraint class, which must be either zero or a number in the range from 101 to 200. The application may use this attribute to distinguish between cutting plane constraints of different classes.[5]

The parameter `flags` currently is not used and must be zero.

[5] Constraint classes numbered from 1 to 100 are reserved for GLPK cutting plane generators.

Ordinal numbers of structural variables (i.e. column indices) $j \in J$ and numerical values of corresponding constraint coefficients a_j must be placed in locations `ind[1]`, ..., `ind[len]` and `val[1]`, ..., `val[len]`, respectively, where `len` $= |J|$ is the number of constraint coefficients, $0 \leq$ `len` $\leq n$, and n is the number of columns in the problem object. Coefficients with identical column indices are not allowed. Zero coefficients are allowed, however, they are ignored.

The parameter `type` specifies the constraint type as follows:

`GLP_LO` means inequality constraint $\Sigma a_j x_j \geq b$;

`GLP_UP` means inequality constraint $\Sigma a_j x_j \leq b$;

The parameter `rhs` specifies the right-hand side b.

All cutting plane constraints in the cut pool are identified by their ordinal numbers 1, 2, ..., *size*, where *size* is the current size of the cut pool. New constraints are always added to the end of the cut pool, thus, ordinal numbers of previously added constraints are not changed.

Returns

The routine `glp_ios_add_row` returns the ordinal number of the cutting plane constraint added, which is the new size of the cut pool.

Example

```
/* generate triangle cutting plane:
   x[i] + x[j] + x[k] <= 1 */
. . .
/* add the constraint to the cut pool */
ind[1] = i, val[1] = 1.0;
ind[2] = j, val[2] = 1.0;
ind[3] = k, val[3] = 1.0;
glp_ios_add_row(tree, NULL, TRIANGLE_CUT, 0, 3, ind, val,
                GLP_UP, 1.0);
```

Comments

Cutting plane constraints added to the cut pool are intended to be then added only to the *current* subproblem, so these constraints can be globally as well as locally valid. However, adding a constraint to the cut pool does not mean that it will be added to the current subproblem—it depends on

the solver's decision: if the constraint seems to be efficient, it is moved from the pool to the current subproblem, otherwise it is simply dropped.[6]

Normally, every time the callback routine is called for cut generation, the cut pool is empty. On the other hand, the solver itself can generate cutting plane constraints (like Gomory's or mixed integer rounding cuts), in which case the cut pool may be non-empty.

5.4.3 glp_ios_del_row—remove constraint from the cut pool

Synopsis

```
void glp_ios_del_row(glp_tree *tree, int i);
```

Description

The routine `glp_ios_del_row` deletes i-th row (cutting plane constraint) from the cut pool, where $1 \leq i \leq size$ is the ordinal number of the constraint in the pool, $size$ is the current size of the cut pool.

Note that deleting a constraint from the cut pool leads to changing ordinal numbers of other constraints remaining in the pool. New ordinal numbers of the remaining constraints are assigned under assumption that the original order of constraints is not changed. Let, for example, there be four constraints a, b, c and d in the cut pool, which have ordinal numbers 1, 2, 3 and 4, respectively, and let constraint b have been deleted. Then after deletion the remaining constraint a, c and d are assigned new ordinal numbers 1, 2 and 3, respectively.

To find the constraint to be deleted the routine `glp_ios_del_row` uses "smart" linear search, so it is recommended to remove constraints in a natural or reverse order and avoid removing them in a random order.

Example

```
/* keep first 10 constraints in the cut pool and remove other
   constraints */
while (glp_ios_pool_size(tree) > 10)
   glp_ios_del_row(tree, glp_ios_pool_size(tree));
```

[6] Globally valid constraints could be saved and then re-used for other subproblems, but currently such feature is not implemented.

5.4.4 glp_ios_clear_pool—remove all constraints from the cut pool

Synopsis

```
void glp_ios_clear_pool(glp_tree *tree);
```

Description

The routine `glp_ios_clear_pool` makes the cut pool empty deleting all existing rows (cutting plane constraints) from it.

Chapter 6

Graph and Network API Routines

6.1 Introduction

6.1.1 Graph program object

In GLPK the base program object used to represent graphs and networks is a directed graph (digraph).

Formally, *digraph* (or simply, *graph*) is a pair $G = (V, A)$, where V is a set of *vertices*, and A is a set *arcs*.[1] Each arc $a \in A$ is an ordered pair of vertices $a = (x, y)$, where $x \in V$ is called *tail vertex* of arc a, and $y \in V$ is called its *head vertex*.

Representation of a graph in the program includes three structs defined by typedef in the header `glpk.h`:

- `glp_graph`, which represents the graph in a whole,
- `glp_vertex`, which represents a vertex of the graph, and
- `glp_arc`, which represents an arc of the graph.

All these three structs are "semi-opaque", i.e. the application program can directly access their fields through pointers, however, changing the fields directly is not allowed—all changes should be performed only with appropriate GLPK API routines.

[1] A may be a multiset.

glp_graph. The struct `glp_graph` has the following fields available to the application program:

`char *name;`
Symbolic name assigned to the graph. It is a pointer to a null terminated character string of length from 1 to 255 characters. If no name is assigned to the graph, this field contains `NULL`.

`int nv;`
The number of vertices in the graph, $nv \geq 0$.

`int na;`
The number of arcs in the graph, $na \geq 0$.

`glp_vertex **v;`
Pointer to an array containing the list of vertices. Element $v[0]$ is not used. Element $v[i]$, $1 \leq i \leq nv$, is a pointer to i-th vertex of the graph. Note that on adding new vertices to the graph the field v may be altered due to reallocation. However, pointers $v[i]$ are not changed while corresponding vertices exist in the graph.

`int v_size;`
Size of vertex data blocks, in bytes, $0 \leq v\_size \leq 256$. (See also the field `data` in the struct `glp_vertex`.)

`int a_size;`
Size of arc data blocks, in bytes, $0 \leq v\_size \leq 256$. (See also the field `data` in the struct `glp_arc`.)

glp_vertex. The struct `glp_vertex` has the following fields available to the application program:

`int i;`
Ordinal number of the vertex, $1 \leq i \leq nv$. Note that element $v[i]$ in the struct `glp_graph` points to the vertex, whose ordinal number is i.

`char *name;`
Symbolic name assigned to the vertex. It is a pointer to a null terminated character string of length from 1 to 255 characters. If no name is assigned to the vertex, this field contains `NULL`.

`void *data;`
Pointer to a data block associated with the vertex. This data block is automatically allocated on creating a new vertex and freed on deleting

the vertex. If $v\_size = 0$, the block is not allocated, and this field contains `NULL`.

`void *temp;`
: Working pointer, which may be used freely for any purposes. The application program can change this field directly.

`glp_arc *in;`
: Pointer to the (unordered) list of incoming arcs. If the vertex has no incoming arcs, this field contains `NULL`.

`glp_arc *out;`
: Pointer to the (unordered) list of outgoing arcs. If the vertex has no outgoing arcs, this field contains `NULL`.

glp_arc. The struct `glp_arc` has the following fields available to the application program:

`glp_vertex *tail;`
: Pointer to a vertex, which is tail endpoint of the arc.

`glp_vertex *head;`
: Pointer to a vertex, which is head endpoint of the arc.

`void *data;`
: Pointer to a data block associated with the arc. This data block is automatically allocated on creating a new arc and freed on deleting the arc. If $v\_size = 0$, the block is not allocated, and this field contains `NULL`.

`void *temp;`
: Working pointer, which may be used freely for any purposes. The application program can change this field directly.

`glp_arc *t_next;`
: Pointer to another arc, which has the same tail endpoint as this one. `NULL` in this field indicates the end of the list of outgoing arcs.

`glp_arc *h_next;`
: Pointer to another arc, which has the same head endpoint as this one. `NULL` in this field indicates the end of the list of incoming arcs.

6.2 Basic graph routines

6.2.1 glp_create_graph—create graph

Synopsis

```
glp_graph *glp_create_graph(int v_size, int a_size);
```

Description

The routine `glp_create_graph` creates a new graph, which initially is empty, i.e. has no vertices and arcs.

The parameter `v_size` specifies the size of vertex data blocks, in bytes, $0 \leq v\_size \leq 256$.

The parameter `a_size` specifies the size of arc data blocks, in bytes, $0 \leq a\_size \leq 256$.

Returns

The routine returns a pointer to the graph created.

6.2.2 glp_set_graph_name—assign (change) graph name

Synopsis

```
void glp_set_graph_name(glp_graph *G, const char *name);
```

Description

The routine `glp_set_graph_name` assigns a symbolic name specified by the character string `name` (1 to 255 chars) to the graph.

If the parameter `name` is `NULL` or an empty string, the routine erases the existing symbolic name of the graph.

6.2.3 glp_add_vertices—add new vertices to graph

Synopsis

```
int glp_add_vertices(glp_graph *G, int nadd);
```

Description

The routine `glp_add_vertices` adds `nadd` vertices to the specified graph. New vertices are always added to the end of the vertex list, so ordinal numbers of existing vertices remain unchanged. Note that this operation may change the field `v` in the struct `glp_graph` (pointer to the vertex array) due to reallocation.

Being added each new vertex is isolated, i.e. has no incident arcs.

If the size of vertex data blocks specified on creating the graph is non-zero, the routine also allocates a memory block of that size for each new vertex added, fills it by binary zeros, and stores a pointer to it in the field `data` of the struct `glp_vertex`. Otherwise, if the block size is zero, the field `data` is set to `NULL`.

Returns

The routine `glp_add_vertices` returns the ordinal number of the first new vertex added to the graph.

6.2.4 glp_set_vertex_name—assign (change) vertex name

Synopsis

```
void glp_set_vertex_name(glp_graph *G, int i, const char *name);
```

Description

The routine `glp_set_vertex_name` assigns a given symbolic name (1 up to 255 characters) to `i`-th vertex of the specified graph.

If the parameter `name` is `NULL` or empty string, the routine erases an existing name of `i`-th vertex.

6.2.5 glp_add_arc—add new arc to graph

Synopsis

```
glp_arc *glp_add_arc(glp_graph *G, int i, int j);
```

Description

The routine `glp_add_arc` adds one new arc to the specified graph.

The parameters `i` and `j` specify the ordinal numbers of, resp., tail and head endpoints (vertices) of the arc. Note that self-loops and multiple arcs are allowed.

If the size of arc data blocks specified on creating the graph is non-zero, the routine also allocates a memory block of that size, fills it by binary zeros, and stores a pointer to it in the field `data` of the struct `glp_arc`. Otherwise, if the block size is zero, the field `data` is set to `NULL`.

6.2.6 glp_del_vertices—delete vertices from graph

Synopsis

```
void glp_del_vertices(glp_graph *G, int ndel, const int num[]);
```

Description

The routine `glp_del_vertices` deletes vertices along with all incident arcs from the specified graph. Ordinal numbers of vertices to be deleted should be placed in locations `num[1]`, ..., `num[ndel]`, `ndel` > 0.

Note that deleting vertices involves changing ordinal numbers of other vertices remaining in the graph. New ordinal numbers of the remaining vertices are assigned under the assumption that the original order of vertices is not changed.

6.2.7 glp_del_arc—delete arc from graph

Synopsis

```
void glp_del_arc(glp_graph *G, glp_arc *a);
```

Description

The routine `glp_del_arc` deletes an arc from the specified graph. The arc to be deleted must exist.

6.2.8 glp_erase_graph—erase graph content

Synopsis

```
void glp_erase_graph(glp_graph *G, int v_size, int a_size);
```

Description

The routine `glp_erase_graph` erases the content of the specified graph. The effect of this operation is the same as if the graph would be deleted with the routine `glp_delete_graph` and then created anew with the routine `glp_create_graph`, with exception that the handle (pointer) to the graph remains valid.

The parameters `v_size` and `a_size` have the same meaning as for the routine `glp_create_graph`.

6.2.9 glp_delete_graph—delete graph

Synopsis

```
void glp_delete_graph(glp_graph *G);
```

Description

The routine `glp_delete_graph` deletes the specified graph and frees all the memory allocated to this program object.

6.3 Graph searching routines

6.3.1 glp_create_v_index—create vertex name index

Synopsis

```
void glp_create_v_index(glp_graph *G);
```

Description

The routine `glp_create_v_index` creates the name index for the specified graph. The name index is an auxiliary data structure, which is intended to quickly (i.e. for logarithmic time) find vertices by their names.

This routine can be called at any time. If the name index already exists, the routine does nothing.

6.3.2 glp_find_vertex—find vertex by its name

Synopsis

```
int glp_find_vertex(glp_graph *G, const char *name);
```

Returns

The routine `glp_find_vertex` returns the ordinal number of a vertex, which is assigned (by the routine `glp_set_vertex_name`) the specified symbolic `name`. If no such vertex exists, the routine returns 0.

6.3.3 glp_delete_v_index—delete vertex name index

Synopsis

```
void glp_delete_v_index(glp_graph *G);
```

Description

The routine `glp_delete_v_index` deletes the name index previously created by the routine `glp_create_v_index` and frees the memory allocated to this auxiliary data structure.

This routine can be called at any time. If the name index does not exist, the routine does nothing.

6.4 Graph reading/writing routines

6.4.1 glp_read_graph—read graph from plain text file

Synopsis

```
int glp_read_graph(glp_graph *G, const char *fname);
```

Description

The routine `glp_read_graph` reads a graph from a plain text file, whose name is specified by the parameter `fname`. Note that before reading data the current content of the graph object is completely erased with the routine `glp_erase_graph`.

For the file format see description of the routine `glp_write_graph`.

Returns

If the operation was successful, the routine returns zero. Otherwise it prints an error message and returns non-zero.

6.4.2 glp_write_graph—write graph to plain text file

Synopsis

```
int glp_write_graph(glp_graph *G, const char *fname);
```

Description

The routine `glp_write_graph` writes the graph to a plain text file, whose name is specified by the parameter `fname`.

Returns

If the operation was successful, the routine returns zero. Otherwise it prints an error message and returns non-zero.

File format

The file created by the routine `glp_write_graph` is a plain text file, which contains the following information:

```
nv na
i[1] j[1]
i[2] j[2]
. . .
i[na] j[na]
```

where:
`nv` is the number of vertices (nodes);
`na` is the number of arcs;
`i[k]`, $k = 1, \ldots, na$, is the index of tail vertex of arc k;
`j[k]`, $k = 1, \ldots, na$, is the index of head vertex of arc k.

6.4.3 glp_read_ccdata—read graph from text file in DIMACS clique/coloring format

Synopsis

```
int glp_read_ccdata(glp_graph *G, int v_wgt,
      const char *fname);
```

Description

The routine *glp_read_ccdata* reads a graph from a text file in DIMACS clique/coloring format. (Though this format is originally designed to represent data for the minimal vertex coloring and maximal clique problems, it may be used to represent general undirected and directed graphs, because the routine allows reading self-loops and multiple edges/arcs keeping the order of vertices specified for each edge/arc of the graph.)

The parameter G specifies the graph object to be read in. Note that before reading data the current content of the graph object is completely erased with the routine *glp_erase_graph*.

The parameter *v_wgt* specifies an offset of the field of type **double** in the vertex data block, to which the routine stores the vertex weight. If *v_wgt* < 0, the vertex weights are not stored.

The character string *fname* specifies the name of a text file to be read in. (If the file name ends with the suffix '`.gz`', the file is assumed to be compressed, in which case the routine decompresses it "on the fly".)

Returns

If the operation was successful, the routine returns zero. Otherwise, it prints an error message and returns non-zero.

DIMACS clique/coloring format[2]

The DIMACS input file is a plain ASCII text file. It contains *lines* of several types described below. A line is terminated with an end-of-line character. Fields in each line are separated by at least one blank space. Each line begins with a one-character designator to identify the line type.

Note that DIMACS requires all numerical quantities to be integers in the range $[-2^{31}, 2^{31} - 1]$ while GLPK allows the quantities to be floating-point numbers.

Comment lines. Comment lines give human-readable information about the file and are ignored by programs. Comment lines can appear anywhere in the file. Each comment line begins with a lower-case character `c`.

```
c This is a comment line
```

Problem line. There is one problem line per data file. The problem line must appear before any node or edge descriptor lines. It has the following format:

```
p edge NODES EDGES
```

The lower-case letter `p` signifies that this is a problem line. The four-character problem designator `edge` identifies the file as containing data for the minimal vertex coloring or maximal clique problem. The `NODES` field contains an integer value specifying the number of vertices in the graph. The `EDGES` field contains an integer value specifying the number of edges (arcs) in the graph.

Vertex descriptors. These lines give the weight assigned to a vertex of the graph. There is one vertex descriptor line for each vertex, with the following format. Vertices without a descriptor take on a default value of 1.

```
n ID VALUE
```

The lower-case character `n` signifies that this is a vertex descriptor line. The `ID` field gives a vertex identification number, an integer between 1 and n, where n is the number of vertices in the graph. The `VALUE` field gives a vertex weight, which can either positive or negative (or zero).

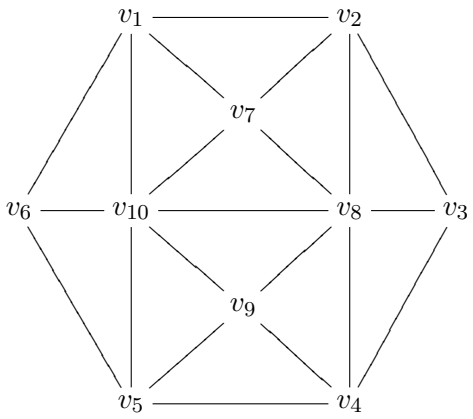
[2]This material is based on the paper "Clique and Coloring Problems Graph Format", which is publically available at `http://dimacs.rutgers.edu/Challenges/`.

Edge descriptors. There is one edge descriptor line for each edge (arc) of the graph, each with the following format:

```
e I J
```

The lower-case character `e` signifies that this is an edge descriptor line. For an edge (arc) (i, j) the fields `I` and `J` specify its endpoints.

Example. The following example of an undirected graph:



might be coded in DIMACS clique/coloring format as follows:

```
c sample.col
c
c This is an example of the vertex coloring problem data
c in DIMACS format.
c
p edge 10 21
c
e 1 2
e 1 6
e 1 7
e 1 10
e 2 3
e 2 7
e 2 8
e 3 4
e 3 8
e 4 5
e 4 8
e 4 9
```

```
e 5 6
e 5 9
e 5 10
e 6 10
e 7 8
e 7 10
e 8 9
e 8 10
e 9 10
c
c eof
```

6.4.4 glp_write_ccdata—write graph to text file in DIMACS clique/coloring format

Synopsis

```
int glp_write_ccdata(glp_graph *G, int v_wgt,
      const char *fname);
```

Description

The routine *glp_write_ccdata* writes the graph object specified by the parameter G to a text file in DIMACS clique/coloring format. (Though this format is originally designed to represent data for the minimal vertex coloring and maximal clique problems, it may be used to represent general undirected and directed graphs, because the routine allows writing self-loops and multiple edges/arcs keeping the order of vertices specified for each edge/arc of the graph.)

The parameter *v_wgt* specifies an offset of the field of type **double** in the vertex data block, which contains the vertex weight. If $v\_wgt < 0$, it is assumed that the weight of each vertex is 1.

The character string *fname* specifies a name of the text file to be written out. (If the file name ends with suffix '`.gz`', the file is assumed to be compressed, in which case the routine performs automatic compression on writing it.)

Returns

If the operation was successful, the routine returns zero. Otherwise, it prints an error message and returns non-zero.

6.5 Graph analysis routines

6.5.1 glp_weak_comp—find all weakly connected components of graph

Synopsis

```
int glp_weak_comp(glp_graph *G, int v_num);
```

Description

The routine `glp_weak_comp` finds all weakly connected components of the specified graph.

The parameter `v_num` specifies an offset of the field of type `int` in the vertex data block, to which the routine stores the number of a weakly connected component containing that vertex. If `v_num` < 0, no component numbers are stored.

The components are numbered in arbitrary order from 1 to `nc`, where `nc` is the total number of components found, $0 \leq$ `nc` $\leq |V|$.

Returns

The routine returns `nc`, the total number of components found.

6.5.2 glp_strong_comp—find all strongly connected components of graph

Synopsis

```
int glp_strong_comp(glp_graph *G, int v_num);
```

Description

The routine `glp_strong_comp` finds all strongly connected components of the specified graph.

The parameter `v_num` specifies an offset of the field of type `int` in the vertex data block, to which the routine stores the number of a strongly connected component containing that vertex. If `v_num` < 0, no component numbers are stored.

The components are numbered in arbitrary order from 1 to `nc`, where `nc` is the total number of components found, $0 \leq$ `nc` $\leq |V|$. However, the component numbering has the property that for every arc $(i \rightarrow j)$ in the graph the condition $num(i) \geq num(j)$ holds.

Returns

The routine returns `nc`, the total number of components found.

References

I. S. Duff, J. K. Reid, Algorithm 529: Permutations to block triangular form, ACM Trans. on Math. Softw. 4 (1978), 189-92.

Example

The following program reads a graph from a plain text file '`graph.txt`' and finds all its strongly connected components.

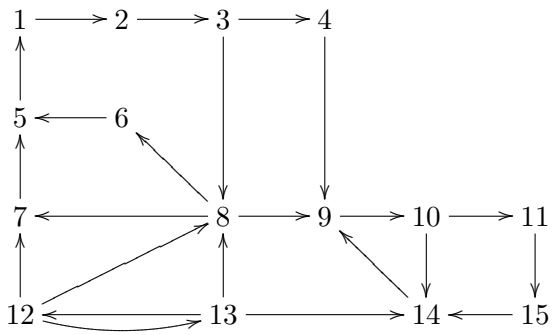
```
#include <stddef.h>
#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

typedef struct { int num; } v_data;

#define vertex(v) ((v_data *)((v)->data))

int main(void)
{     glp_graph *G;
      int i, nc;
      G = glp_create_graph(sizeof(v_data), 0);
      glp_read_graph(G, "graph.txt");
      nc = glp_strong_comp(G, offsetof(v_data, num));
      printf("nc = %d\n", nc);
      for (i = 1; i <= G->nv; i++)
         printf("num[%d] = %d\n", i, vertex(G->v[i])->num);
      glp_delete_graph(G);
      return 0;
}
```

If the file '`graph.txt`' contains the graph shown below:



the program output may look like follows:

```
Reading graph from ‘graph.txt’...
Graph has 15 vertices and 30 arcs
31 lines were read
nc = 4
num[1] = 3
num[2] = 3
num[3] = 3
num[4] = 2
num[5] = 3
num[6] = 3
num[7] = 3
num[8] = 3
num[9] = 1
num[10] = 1
num[11] = 1
num[12] = 4
num[13] = 4
num[14] = 1
num[15] = 1
```

6.6 Minimum cost flow problem

6.6.1 Background

The *minimum cost flow problem* (MCFP) is stated as follows. Let there be given a directed graph (flow network) $G = (V, A)$, where V is a set of vertices (nodes), and $A \subseteq V \times V$ is a set of arcs. Let for each node $i \in V$ there be given a quantity b_i having the following meaning:

if $b_i > 0$, then $|b_i|$ is a *supply* at node i, which shows how many flow units are *generated* at node i (or, equivalently, entering the network through node i from the outside);

if $b_i < 0$, then $|b_i|$ is a *demand* at node i, which shows how many flow units are *lost* at node i (or, equivalently, leaving the network through node i to the outside);

if $b_i = 0$, then i is a *transshipment* node, at which the flow is conserved, i.e. neither generated nor lost.

Let also for each arc $a = (i, j) \in A$ there be given the following three quantities:

l_{ij}, a (non-negative) lower bound to the flow through arc (i, j);

u_{ij}, an upper bound to the flow through arc (i, j), which is the *arc capacity*;

c_{ij}, a per-unit cost of the flow through arc (i, j).

The problem is to find flows x_{ij} through every arc of the network, which satisfy the specified bounds and the conservation constraints at all nodes, and minimize the total flow cost. Here the conservation constraint at a node means that the total flow entering this node through its incoming arcs plus the supply at this node must be equal to the total flow leaving this node through its outgoing arcs plus the demand at this node.

An example of the minimum cost flow problem is shown on Fig. 1.

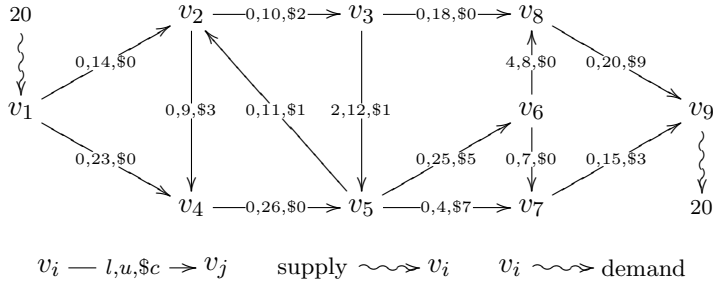


Fig. 1. An example of the minimum cost flow problem.

The minimum cost flow problem can be naturally formulated as the following LP problem:

minimize

$$z = \sum_{(i,j)\in A} c_{ij}x_{ij} \tag{1}$$

subject to

$$\sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} = b_i \quad \text{for all } i \in V \tag{2}$$

$$l_{ij} \le x_{ij} \le u_{ij} \quad \text{for all } (i,j) \in A \tag{3}$$

6.6.2 glp_read_mincost—read minimum cost flow problem data in DIMACS format

Synopsis

```
int glp_read_mincost(glp_graph *G, int v_rhs, int a_low,
      int a_cap, int a_cost, const char *fname);
```

Description

The routine `glp_read_mincost` reads the minimum cost flow problem data from a text file in DIMACS format.

The parameter `G` specifies the graph object, to which the problem data have to be stored. Note that before reading data the current content of the graph object is completely erased with the routine `glp_erase_graph`.

The parameter `v_rhs` specifies an offset of the field of type `double` in the vertex data block, to which the routine stores b_i, the supply/demand value. If `v_rhs` < 0, the value is not stored.

The parameter `a_low` specifies an offset of the field of type `double` in the arc data block, to which the routine stores l_{ij}, the lower bound to the arc flow. If `a_low` < 0, the lower bound is not stored.

The parameter `a_cap` specifies an offset of the field of type `double` in the arc data block, to which the routine stores u_{ij}, the upper bound to the arc flow (the arc capacity). If `a_cap` < 0, the upper bound is not stored.

The parameter `a_cost` specifies an offset of the field of type `double` in the arc data block, to which the routine stores c_{ij}, the per-unit cost of the arc flow. If `a_cost` < 0, the cost is not stored.

The character string `fname` specifies the name of a text file to be read in. (If the file name name ends with the suffix '`.gz`', the file is assumed to be compressed, in which case the routine decompresses it "on the fly".)

Returns

If the operation was successful, the routine returns zero. Otherwise, it prints an error message and returns non-zero.

Example

```
typedef struct
{     /* vertex data block */
      ...
      double rhs;
      ...
} v_data;

typedef struct
{     /* arc data block */
      ...
      double low, cap, cost;
      ...
} a_data;

int main(void)
{     glp_graph *G;
      int ret;
      G = glp_create_graph(sizeof(v_data), sizeof(a_data));
      ret = glp_read_mincost(G, offsetof(v_data, rhs),
         offsetof(a_data, low), offsetof(a_data, cap),
         offsetof(a_data, cost), "sample.min");
      if (ret != 0) goto ...
      ...
}
```

DIMACS minimum cost flow problem format[3]

The DIMACS input file is a plain ASCII text file. It contains *lines* of several types described below. A line is terminated with an end-of-line character. Fields in each line are separated by at least one blank space. Each line begins with a one-character designator to identify the line type.

Note that DIMACS requires all numerical quantities to be integers in the range $[-2^{31},\ 2^{31}-1]$ while GLPK allows the quantities to be floating-point numbers.

[3]This material is based on the paper "The First DIMACS International Algorithm Implementation Challenge: Problem Definitions and Specifications", which is publically available at `http://dimacs.rutgers.edu/Challenges/`.

Comment lines. Comment lines give human-readable information about the file and are ignored by programs. Comment lines can appear anywhere in the file. Each comment line begins with a lower-case character `c`.

```
c This is a comment line
```

Problem line. There is one problem line per data file. The problem line must appear before any node or arc descriptor lines. It has the following format:

```
p min NODES ARCS
```

The lower-case character `p` signifies that this is a problem line. The three-character problem designator `min` identifies the file as containing specification information for the minimum cost flow problem. The `NODES` field contains an integer value specifying the number of nodes in the network. The `ARCS` field contains an integer value specifying the number of arcs in the network.

Node descriptors. All node descriptor lines must appear before all arc descriptor lines. The node descriptor lines describe supply and demand nodes, but not transshipment nodes. That is, only nodes with non-zero node supply/demand values appear. There is one node descriptor line for each such node, with the following format:

```
n ID FLOW
```

The lower-case character `n` signifies that this is a node descriptor line. The `ID` field gives a node identification number, an integer between 1 and `NODES`. The `FLOW` field gives the amount of supply (if positive) or demand (if negative) at node `ID`.

Arc descriptors. There is one arc descriptor line for each arc in the network. Arc descriptor lines are of the following format:

```
a SRC DST LOW CAP COST
```

The lower-case character `a` signifies that this is an arc descriptor line. For a directed arc (i, j) the `SRC` field gives the identification number i for the tail endpoint, and the `DST` field gives the identification number j for the head endpoint. Identification numbers are integers between 1 and `NODES`. The

`LOW` field specifies the minimum amount of flow that can be sent along arc (i, j), and the `CAP` field gives the maximum amount of flow that can be sent along arc (i, j) in a feasible flow. The `COST` field contains the per-unit cost of flow sent along arc (i, j).

Example. Below here is an example of the data file in DIMACS format corresponding to the minimum cost flow problem shown on Fig 1.

```
c sample.min
c
c This is an example of the minimum cost flow problem data
c in DIMACS format.
c
p min 9 14
c
n 1 20
n 9 -20
c
a 1 2 0 14 0
a 1 4 0 23 0
a 2 3 0 10 2
a 2 4 0  9 3
a 3 5 2 12 1
a 3 8 0 18 0
a 4 5 0 26 0
a 5 2 0 11 1
a 5 6 0 25 5
a 5 7 0  4 7
a 6 7 0  7 0
a 6 8 4  8 0
a 7 9 0 15 3
a 8 9 0 20 9
c
c eof
```

6.6.3 glp_write_mincost—write minimum cost flow problem data in DIMACS format

Synopsis

```
int glp_write_mincost(glp_graph *G, int v_rhs, int a_low,
      int a_cap, int a_cost, const char *fname);
```

Description

The routine `glp_write_mincost` writes the minimum cost flow problem data to a text file in DIMACS format.

The parameter `G` is the graph (network) program object, which specifies the minimum cost flow problem instance.

The parameter `v_rhs` specifies an offset of the field of type `double` in the vertex data block, which contains b_i, the supply/demand value. If `v_rhs` < 0, it is assumed that $b_i = 0$ for all nodes.

The parameter `a_low` specifies an offset of the field of type `double` in the arc data block, which contains l_{ij}, the lower bound to the arc flow. If `a_low` < 0, it is assumed that $l_{ij} = 0$ for all arcs.

The parameter `a_cap` specifies an offset of the field of type `double` in the arc data block, which contains u_{ij}, the upper bound to the arc flow (the arc capacity). If the upper bound is specified as `DBL_MAX`, it is assumed that $u_{ij} = \infty$, i.e. the arc is uncapacitated. If `a_cap` < 0, it is assumed that $u_{ij} = 1$ for all arcs.

The parameter `a_cost` specifies an offset of the field of type `double` in the arc data block, which contains c_{ij}, the per-unit cost of the arc flow. If `a_cost` < 0, it is assumed that $c_{ij} = 0$ for all arcs.

The character string `fname` specifies a name of the text file to be written out. (If the file name ends with suffix '`.gz`', the file is assumed to be compressed, in which case the routine performs automatic compression on writing it.)

Returns

If the operation was successful, the routine returns zero. Otherwise, it prints an error message and returns non-zero.

6.6.4 glp_mincost_lp—convert minimum cost flow problem to LP

Synopsis

```
void glp_mincost_lp(glp_prob *lp, glp_graph *G, int names,
      int v_rhs, int a_low, int a_cap, int a_cost);
```

Description

The routine `glp_mincost_lp` builds LP problem (1)—(3), which corresponds to the specified minimum cost flow problem.

The parameter `lp` is the resultant LP problem object to be built. Note that on entry its current content is erased with the routine `glp_erase_prob`.

The parameter `G` is the graph (network) program object, which specifies the minimum cost flow problem instance.

The parameter `names` is a flag. If it is `GLP_ON`, the routine uses symbolic names of the graph object components to assign symbolic names to the LP problem object components. If the flag is `GLP_OFF`, no symbolic names are assigned.

The parameter `v_rhs` specifies an offset of the field of type `double` in the vertex data block, which contains b_i, the supply/demand value. If `v_rhs` < 0, it is assumed that $b_i = 0$ for all nodes.

The parameter `a_low` specifies an offset of the field of type `double` in the arc data block, which contains l_{ij}, the lower bound to the arc flow. If `a_low` < 0, it is assumed that $l_{ij} = 0$ for all arcs.

The parameter `a_cap` specifies an offset of the field of type `double` in the arc data block, which contains u_{ij}, the upper bound to the arc flow (the arc capacity). If the upper bound is specified as `DBL_MAX`, it is assumed that $u_{ij} = \infty$, i.e. the arc is uncapacitated. If `a_cap` < 0, it is assumed that $u_{ij} = 1$ for all arcs.

The parameter `a_cost` specifies an offset of the field of type `double` in the arc data block, which contains c_{ij}, the per-unit cost of the arc flow. If `a_cost` < 0, it is assumed that $c_{ij} = 0$ for all arcs.

Example

The example program below reads the minimum cost problem instance in DIMACS format from file '`sample.min`', converts the instance to LP, and then writes the resultant LP in CPLEX format to file '`mincost.lp`'.

```
#include <stddef.h>
#include <glpk.h>

typedef struct { double rhs; } v_data;
typedef struct { double low, cap, cost; } a_data;

int main(void)
{     glp_graph *G;
      glp_prob *lp;
      G = glp_create_graph(sizeof(v_data), sizeof(a_data));
      glp_read_mincost(G, offsetof(v_data, rhs),
         offsetof(a_data, low), offsetof(a_data, cap),
         offsetof(a_data, cost), "sample.min");
      lp = glp_create_prob();
      glp_mincost_lp(lp, G, GLP_ON, offsetof(v_data, rhs),
         offsetof(a_data, low), offsetof(a_data, cap),
         offsetof(a_data, cost));
      glp_delete_graph(G);
      glp_write_lp(lp, NULL, "mincost.lp");
      glp_delete_prob(lp);
      return 0;
}
```

If '`sample.min`' is the example data file from the subsection describing the routine `glp_read_mincost`, file '`mincost.lp`' may look like follows:

```
Minimize
 obj: + 3 x(2,4) + 2 x(2,3) + x(3,5) + 7 x(5,7) + 5 x(5,6)
 + x(5,2) + 3 x(7,9) + 9 x(8,9)

Subject To
 r_1: + x(1,2) + x(1,4) = 20
 r_2: - x(5,2) + x(2,3) + x(2,4) - x(1,2) = 0
 r_3: + x(3,5) + x(3,8) - x(2,3) = 0
 r_4: + x(4,5) - x(2,4) - x(1,4) = 0
 r_5: + x(5,2) + x(5,6) + x(5,7) - x(4,5) - x(3,5) = 0
 r_6: + x(6,7) + x(6,8) - x(5,6) = 0
 r_7: + x(7,9) - x(6,7) - x(5,7) = 0
 r_8: + x(8,9) - x(6,8) - x(3,8) = 0
 r_9: - x(8,9) - x(7,9) = -20

Bounds
 0 <= x(1,4) <= 23
 0 <= x(1,2) <= 14
 0 <= x(2,4) <= 9
 0 <= x(2,3) <= 10
 0 <= x(3,8) <= 18
 2 <= x(3,5) <= 12
 0 <= x(4,5) <= 26
```

```
 0 <= x(5,7) <= 4
 0 <= x(5,6) <= 25
 0 <= x(5,2) <= 11
 4 <= x(6,8) <= 8
 0 <= x(6,7) <= 7
 0 <= x(7,9) <= 15
 0 <= x(8,9) <= 20

End
```

6.6.5 glp_mincost_okalg—solve minimum cost flow problem with out-of-kilter algorithm

Synopsis

```
int glp_mincost_okalg(glp_graph *G, int v_rhs, int a_low,
      int a_cap, int a_cost, double *sol, int a_x, int v_pi);
```

Description

The routine `glp_mincost_okalg` finds optimal solution to the minimum cost flow problem with the out-of-kilter algorithm.[4] Note that this routine requires all the problem data to be integer-valued.

The parameter `G` is a graph (network) program object which specifies the minimum cost flow problem instance to be solved.

The parameter `v_rhs` specifies an offset of the field of type `double` in the vertex data block, which contains b_i, the supply/demand value. This value must be integer in the range [$-$`INT_MAX`, $+$`INT_MAX`]. If `v_rhs` < 0, it is assumed that $b_i = 0$ for all nodes.

The parameter `a_low` specifies an offset of the field of type `double` in the arc data block, which contains l_{ij}, the lower bound to the arc flow. This bound must be integer in the range [0, `INT_MAX`]. If `a_low` < 0, it is assumed that $l_{ij} = 0$ for all arcs.

The parameter `a_cap` specifies an offset of the field of type `double` in the arc data block, which contains u_{ij}, the upper bound to the arc flow (the arc capacity). This bound must be integer in the range [l_{ij}, `INT_MAX`]. If `a_cap` < 0, it is assumed that $u_{ij} = 1$ for all arcs.

The parameter `a_cost` specifies an offset of the field of type `double` in the arc data block, which contains c_{ij}, the per-unit cost of the arc flow. This

[4] GLPK implementation of the out-of-kilter algorithm is based on the following book: L. R. Ford, Jr., and D. R. Fulkerson, "Flows in Networks," The RAND Corp., Report R-375-PR (August 1962), Chap. III "Minimal Cost Flow Problems," pp. 113-26.

value must be integer in the range [−INT_MAX, +INT_MAX]. If a_cost < 0, it is assumed that $c_{ij} = 0$ for all arcs.

The parameter sol specifies a location, to which the routine stores the objective value (that is, the total cost) found. If sol is NULL, the objective value is not stored.

The parameter a_x specifies an offset of the field of type double in the arc data block, to which the routine stores x_{ij}, the arc flow found. If a_x < 0, the arc flow value is not stored.

The parameter v_pi specifies an offset of the field of type double in the vertex data block, to which the routine stores π_i, the node potential, which is the Lagrange multiplier for the corresponding flow conservation equality constraint (see (2) in Subsection "Background"). If necessary, the application program may use the node potentials to compute λ_{ij}, reduced costs of the arc flows x_{ij}, which are the Lagrange multipliers for the arc flow bound constraints (see (3) ibid.), using the following formula:

$$\lambda_{ij} = c_{ij} - (\pi_i - \pi_j),$$

where c_{ij} is the per-unit cost for arc (i, j).

Note that all solution components (the objective value, arc flows, and node potentials) computed by the routine are always integer-valued.

Returns

0	Optimal solution found.
GLP_ENOPFS	No (primal) feasible solution exists.
GLP_EDATA	Unable to start the search, because some problem data are either not integer-valued or out of range. This code is also returned if the total supply, which is the sum of b_i over all source nodes (nodes with $b_i > 0$), exceeds INT_MAX.
GLP_ERANGE	The search was prematurely terminated because of integer overflow.
GLP_EFAIL	An error has been detected in the program logic. (If this code is returned for your problem instance, please report to <bug-glpk@gnu.org>.)

Comments

By design the out-of-kilter algorithm is applicable only to networks, where $b_i = 0$ for *all* nodes, i.e. actually this algorithm finds a minimal cost *circulation*. Due to this requirement the routine glp_mincost_okalg converts

the original network to a network suitable for the out-of-kilter algorithm in the following way:[5]

1) it adds two auxiliary nodes s and t;

2) for each original node i with $b_i > 0$ the routine adds auxiliary supply arc $(s \rightarrow i)$, flow x_{si} through which is costless ($c_{si} = 0$) and fixed to $+b_i$ ($l_{si} = u_{si} = +b_i$);

3) for each original node i with $b_i < 0$ the routine adds auxiliary demand arc $(i \rightarrow t)$, flow x_{it} through which is costless ($c_{it} = 0$) and fixed to $-b_i$ ($l_{it} = u_{it} = -b_i$);

4) finally, the routine adds auxiliary feedback arc $(t \rightarrow s)$, flow x_{ts} through which is also costless ($c_{ts} = 0$) and fixed to F ($l_{ts} = u_{ts} = F$), where $F = \sum_{b_i > 0} b_i$ is the total supply.

Example

The example program below reads the minimum cost problem instance in DIMACS format from file '`sample.min`', solves it by using the routine `glp_mincost_okalg`, and writes the solution found to the standard output.

```
#include <stddef.h>
#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

typedef struct { double rhs, pi; } v_data;
typedef struct { double low, cap, cost, x; } a_data;

#define node(v) ((v_data *)((v)->data))
#define arc(a)  ((a_data *)((a)->data))

int main(void)
{     glp_graph *G;
      glp_vertex *v, *w;
      glp_arc *a;
      int i, ret;
      double sol;
      G = glp_create_graph(sizeof(v_data), sizeof(a_data));
      glp_read_mincost(G, offsetof(v_data, rhs),
         offsetof(a_data, low), offsetof(a_data, cap),
         offsetof(a_data, cost), "sample.min");
      ret = glp_mincost_okalg(G, offsetof(v_data, rhs),
         offsetof(a_data, low), offsetof(a_data, cap),
```

[5] The conversion is performed internally and does not change the original network program object passed to the routine.

```
         offsetof(a_data, cost), &sol, offsetof(a_data, x),
         offsetof(v_data, pi));
      printf("ret = %d; sol = %5g\n", ret, sol);
      for (i = 1; i <= G->nv; i++)
      {  v = G->v[i];
         printf("node %d:    pi = %5g\n", i, node(v)->pi);
         for (a = v->out; a != NULL; a = a->t_next)
         {  w = a->head;
            printf("arc  %d->%d: x  = %5g; lambda = %5g\n",
               v->i, w->i, arc(a)->x,
               arc(a)->cost - (node(v)->pi - node(w)->pi));
         }
      }
      glp_delete_graph(G);
      return 0;
}
```

If '`sample.min`' is the example data file from the subsection describing the routine `glp_read_mincost`, the output may look like follows:

```
Reading min-cost flow problem data from 'sample.min'...
Flow network has 9 nodes and 14 arcs
24 lines were read
ret = 0; sol =   213
node 1:    pi =   -12
arc  1->4: x  =    13; lambda =     0
arc  1->2: x  =     7; lambda =     0
node 2:    pi =   -12
arc  2->4: x  =     0; lambda =     3
arc  2->3: x  =     7; lambda =     0
node 3:    pi =   -14
arc  3->8: x  =     5; lambda =     0
arc  3->5: x  =     2; lambda =     3
node 4:    pi =   -12
arc  4->5: x  =    13; lambda =     0
node 5:    pi =   -12
arc  5->7: x  =     4; lambda =    -1
arc  5->6: x  =    11; lambda =     0
arc  5->2: x  =     0; lambda =     1
node 6:    pi =   -17
arc  6->8: x  =     4; lambda =     3
arc  6->7: x  =     7; lambda =    -3
node 7:    pi =   -20
arc  7->9: x  =    11; lambda =     0
node 8:    pi =   -14
arc  8->9: x  =     9; lambda =     0
node 9:    pi =   -23
```

6.6.6 glp_netgen—Klingman's network problem generator

Synopsis

```
int glp_netgen(glp_graph *G, int v_rhs, int a_cap, int a_cost,
      const int parm[1+15]);
```

Description

The routine `glp_netgen` is a GLPK version of the network problem generator developed by Dr. Darwin Klingman.[6] It can create capacitated and uncapacitated minimum cost flow (or transshipment), transportation, and assignment problems.

The parameter `G` specifies the graph object, to which the generated problem data have to be stored. Note that on entry the graph object is erased with the routine `glp_erase_graph`.

The parameter `v_rhs` specifies an offset of the field of type `double` in the vertex data block, to which the routine stores the supply or demand value. If `v_rhs` < 0, the value is not stored.

The parameter `a_cap` specifies an offset of the field of type `double` in the arc data block, to which the routine stores the arc capacity. If `a_cap` < 0, the capacity is not stored.

The parameter `a_cost` specifies an offset of the field of type `double` in the arc data block, to which the routine stores the per-unit cost if the arc flow. If `a_cost` < 0, the cost is not stored.

The array `parm` contains description of the network to be generated:

`parm[0]`		not used
`parm[1]`	`iseed`	8-digit positive random number seed
`parm[2]`	`nprob`	8-digit problem id number
`parm[3]`	`nodes`	total number of nodes
`parm[4]`	`nsorc`	total number of source nodes (including transshipment nodes)
`parm[5]`	`nsink`	total number of sink nodes (including transshipment nodes)
`parm[6]`	`iarcs`	number of arc
`parm[7]`	`mincst`	minimum cost for arcs
`parm[8]`	`maxcst`	maximum cost for arcs
`parm[9]`	`itsup`	total supply

[6] D. Klingman, A. Napier, and J. Stutz. NETGEN: A program for generating large scale capacitated assignment, transportation, and minimum cost flow networks. Management Science 20 (1974), 814-20.

parm[10]	ntsorc	number of transshipment source nodes
parm[11]	ntsink	number of transshipment sink nodes
parm[12]	iphic	percentage of skeleton arcs to be given the maximum cost
parm[13]	ipcap	percentage of arcs to be capacitated
parm[14]	mincap	minimum upper bound for capacitated arcs
parm[15]	maxcap	maximum upper bound for capacitated arcs

Notes

1. The routine generates a transportation problem if:

$$\texttt{nsorc} + \texttt{nsink} = \texttt{nodes},\ \texttt{ntsorc} = 0,\ \text{and}\ \texttt{ntsink} = 0.$$

2. The routine generates an assignment problem if the requirements for a transportation problem are met and:

$$\texttt{nsorc} = \texttt{nsink}\ \text{and}\ \texttt{itsup} = \texttt{nsorc}.$$

3. The routine always generates connected graphs. So, if the number of requested arcs has been reached and the generated instance is not fully connected, the routine generates a few remaining arcs to ensure connectedness. Thus, the actual number of arcs generated by the routine may be greater than the requested number of arcs.

Returns

If the instance was successfully generated, the routine `glp_netgen` returns zero; otherwise, if specified parameters are inconsistent, the routine returns a non-zero error code.

6.6.7 glp_gridgen—grid-like network problem generator

Synopsis

```
int glp_gridgen(glp_graph *G, int v_rhs, int a_cap, int a_cost,
      const int parm[1+14]);
```

Description

The routine `glp_gridgen` is a GLPK version of the grid-like network problem generator developed by Yusin Lee and Jim Orlin.[7]

[7] Y. Lee and J. Orlin. GRIDGEN generator., 1991. The original code is publically available from <ftp://dimacs.rutgers.edu/pub/netflow/generators/network/gridgen>.

The parameter `G` specifies the graph object, to which the generated problem data have to be stored. Note that on entry the graph object is erased with the routine `glp_erase_graph`.

The parameter `v_rhs` specifies an offset of the field of type `double` in the vertex data block, to which the routine stores the supply or demand value. If `v_rhs` < 0, the value is not stored.

The parameter `a_cap` specifies an offset of the field of type `double` in the arc data block, to which the routine stores the arc capacity. If `a_cap` < 0, the capacity is not stored.

The parameter `a_cost` specifies an offset of the field of type `double` in the arc data block, to which the routine stores the per-unit cost if the arc flow. If `a_cost` < 0, the cost is not stored.

The array `parm` contains parameters of the network to be generated:

`parm[0]`	not used
`parm[1]`	two-ways arcs indicator: 1 — if links in both direction should be generated 0 — otherwise
`parm[2]`	random number seed (a positive integer)
`parm[3]`	number of nodes (the number of nodes generated might be slightly different to make the network a grid)
`parm[4]`	grid width
`parm[5]`	number of sources
`parm[6]`	number of sinks
`parm[7]`	average degree
`parm[8]`	total flow
`parm[9]`	distribution of arc costs: 1 — uniform 2 — exponential
`parm[10]`	lower bound for arc cost (uniform) 100λ (exponential)
`parm[11]`	upper bound for arc cost (uniform) not used (exponential)
`parm[12]`	distribution of arc capacities: 1 — uniform 2 — exponential
`parm[13]`	lower bound for arc capacity (uniform) 100λ (exponential)
`parm[14]`	upper bound for arc capacity (uniform) not used (exponential)

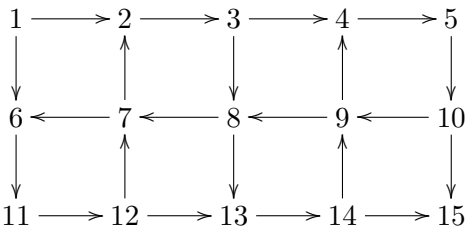
Returns

If the instance was successfully generated, the routine `glp_gridgen` returns zero; otherwise, if specified parameters are inconsistent, the routine returns a non-zero error code.

Comments[8]

This network generator generates a grid-like network plus a super node. In additional to the arcs connecting the nodes in the grid, there is an arc from each supply node to the super node and from the super node to each demand node to guarantee feasiblity. These arcs have very high costs and very big capacities.

The idea of this network generator is as follows: First, a grid of $n_1 \times n_2$ is generated. For example, 5×3. The nodes are numbered as 1 to 15, and the supernode is numbered as $n_1 \times n_2 + 1$. Then arcs between adjacent nodes are generated. For these arcs, the user is allowed to specify either to generate two-way arcs or one-way arcs. If two-way arcs are to be generated, two arcs, one in each direction, will be generated between each adjacent node pairs. Otherwise, only one arc will be generated. If this is the case, the arcs will be generated in alterntive directions as shown below.



Then the arcs between the super node and the source/sink nodes are added as mentioned before. If the number of arcs still doesn't reach the requirement, additional arcs will be added by uniformly picking random node pairs. There is no checking to prevent multiple arcs between any pair of nodes. However, there will be no self-arcs (arcs that poins back to its tail node) in the network.

The source and sink nodes are selected uniformly in the network, and the imbalances of each source/sink node are also assigned by uniform distribution.

[8] This material is based on comments to the original version of GRIDGEN.

6.7 Maximum flow problem

6.7.1 Background

The *maximum flow problem* (MAXFLOW) is stated as follows. Let there be given a directed graph (flow network) $G = (V, A)$, where V is a set of vertices (nodes), and $A \subseteq V \times V$ is a set of arcs. Let also for each arc $a = (i, j) \in A$ there be given its capacity u_{ij}. The problem is, for given *source* node $s \in V$ and *sink* node $t \in V$, to find flows x_{ij} through every arc of the network, which satisfy the specified arc capacities and the conservation constraints at all nodes, and maximize the total flow F through the network from s to t. Here the conservation constraint at a node means that the total flow entering this node through its incoming arcs (plus F, if it is the source node) must be equal to the total flow leaving this node through its outgoing arcs (plus F, if it is the sink node).

An example of the maximum flow problem, where $s = v_1$ and $t = v_9$, is shown on Fig. 2.

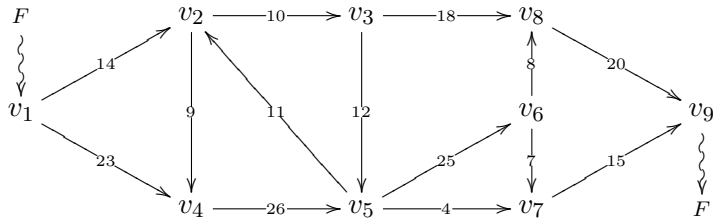


Fig. 2. An example of the maximum flow problem.

The maximum flow problem can be naturally formulated as the following LP problem:

maximize

$$F \tag{4}$$

subject to

$$\sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} = \begin{cases} +F, & \text{for } i = s \\ 0, & \text{for all } i \in V\backslash\{s,t\} \\ -F, & \text{for } i = t \end{cases} \tag{5}$$

$$0 \le x_{ij} \le u_{ij} \quad \text{for all } (i,j) \in A \tag{6}$$

where $F \ge 0$ is an additional variable playing the role of the objective.

Another LP formulation of the maximum flow problem, which does not include the variable F, is the following:

maximize

$$z = \sum_{(s,j)\in A} x_{sj} - \sum_{(j,s)\in A} x_{js} \ (= F) \tag{7}$$

subject to

$$\sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} \begin{cases} \geq 0, & \text{for } i = s \\ = 0, & \text{for all } i \in V\backslash\{s,t\} \\ \leq 0, & \text{for } i = t \end{cases} \tag{8}$$

$$0 \leq x_{ij} \leq u_{ij} \quad \text{for all } (i,j) \in A \tag{9}$$

6.7.2 glp_read_maxflow—read maximum flow problem data in DIMACS format

Synopsis

```
int glp_read_maxflow(glp_graph *G, int *s, int *t, int a_cap,
      const char *fname);
```

Description

The routine `glp_read_maxflow` reads the maximum flow problem data from a text file in DIMACS format.

The parameter `G` specifies the graph object, to which the problem data have to be stored. Note that before reading data the current content of the graph object is completely erased with the routine `glp_erase_graph`.

The pointer `s` specifies a location, to which the routine stores the ordinal number of the source node. If `s` is `NULL`, the source node number is not stored.

The pointer `t` specifies a location, to which the routine stores the ordinal number of the sink node. If `t` is `NULL`, the sink node number is not stored.

The parameter `a_cap` specifies an offset of the field of type `double` in the arc data block, to which the routine stores u_{ij}, the arc capacity. If `a_cap` < 0, the arc capacity is not stored.

The character string `fname` specifies the name of a text file to be read in. (If the file name name ends with the suffix '`.gz`', the file is assumed to be compressed, in which case the routine decompresses it "on the fly".)

Returns

If the operation was successful, the routine returns zero. Otherwise, it prints an error message and returns non-zero.

Example

```
typedef struct
{     /* arc data block */
      ...
      double cap;
      ...
} a_data;

int main(void)
{     glp_graph *G;
      int s, t, ret;
      G = glp_create_graph(..., sizeof(a_data));
      ret = glp_read_maxflow(G, &s, &t, offsetof(a_data, cap),
         "sample.max");
      if (ret != 0) goto ...
      ...
}
```

DIMACS maximum flow problem format[9]

The DIMACS input file is a plain ASCII text file. It contains *lines* of several types described below. A line is terminated with an end-of-line character. Fields in each line are separated by at least one blank space. Each line begins with a one-character designator to identify the line type.

Note that DIMACS requires all numerical quantities to be integers in the range $[-2^{31},\ 2^{31}-1]$ while GLPK allows the quantities to be floating-point numbers.

Comment lines. Comment lines give human-readable information about the file and are ignored by programs. Comment lines can appear anywhere in the file. Each comment line begins with a lower-case character `c`.

```
c This is a comment line
```

[9] This material is based on the paper "The First DIMACS International Algorithm Implementation Challenge: Problem Definitions and Specifications", which is publically available at `http://dimacs.rutgers.edu/Challenges/`.

Problem line. There is one problem line per data file. The problem line must appear before any node or arc descriptor lines. It has the following format:

```
p max NODES ARCS
```

The lower-case character `p` signifies that this is a problem line. The three-character problem designator `max` identifies the file as containing specification information for the maximum flow problem. The `NODES` field contains an integer value specifying the number of nodes in the network. The `ARCS` field contains an integer value specifying the number of arcs in the network.

Node descriptors. Two node descriptor lines for the source and sink nodes must appear before all arc descriptor lines. They may appear in either order, each with the following format:

```
n ID WHICH
```

The lower-case character `n` signifies that this a node descriptor line. The `ID` field gives a node identification number, an integer between 1 and `NODES`. The `WHICH` field gives either a lower-case `s` or `t`, designating the source and sink, respectively.

Arc descriptors. There is one arc descriptor line for each arc in the network. Arc descriptor lines are of the following format:

```
a SRC DST CAP
```

The lower-case character `a` signifies that this is an arc descriptor line. For a directed arc (i, j) the `SRC` field gives the identification number i for the tail endpoint, and the `DST` field gives the identification number j for the head endpoint. Identification numbers are integers between 1 and `NODES`. The `CAP` field gives the arc capacity, i.e. maximum amount of flow that can be sent along arc (i, j) in a feasible flow.

Example. Below here is an example of the data file in DIMACS format corresponding to the maximum flow problem shown on Fig 2.

```
c sample.max
c
c This is an example of the maximum flow problem data
c in DIMACS format.
c
p max 9 14
c
n 1 s
n 9 t
c
a 1 2 14
a 1 4 23
a 2 3 10
a 2 4  9
a 3 5 12
a 3 8 18
a 4 5 26
a 5 2 11
a 5 6 25
a 5 7  4
a 6 7  7
a 6 8  8
a 7 9 15
a 8 9 20
c
c eof
```

6.7.3 glp_write_maxflow—write maximum flow problem data in DIMACS format

Synopsis

```
int glp_write_maxflow(glp_graph *G, int s, int t, int a_cap,
      const char *fname);
```

Description

The routine `glp_write_maxflow` writes the maximum flow problem data to a text file in DIMACS format.

The parameter `G` is the graph (network) program object, which specifies the maximum flow problem instance.

The parameter `s` specifies the ordinal number of the source node.

The parameter `t` specifies the ordinal number of the sink node.

The parameter `a_cap` specifies an offset of the field of type `double` in the arc data block, which contains u_{ij}, the upper bound to the arc flow (the

arc capacity). If the upper bound is specified as `DBL_MAX`, it is assumed that $u_{ij} = \infty$, i.e. the arc is uncapacitated. If `a_cap` < 0, it is assumed that $u_{ij} = 1$ for all arcs.

The character string `fname` specifies a name of the text file to be written out. (If the file name ends with suffix '`.gz`', the file is assumed to be compressed, in which case the routine performs automatic compression on writing it.)

Returns

If the operation was successful, the routine returns zero. Otherwise, it prints an error message and returns non-zero.

6.7.4 glp_maxflow_lp—convert maximum flow problem to LP

Synopsis

```
void glp_maxflow_lp(glp_prob *lp, glp_graph *G, int names,
      int s, int t, int a_cap);
```

Description

The routine `glp_maxflow_lp` builds LP problem (7)—(9), which corresponds to the specified maximum flow problem.

The parameter `lp` is the resultant LP problem object to be built. Note that on entry its current content is erased with the routine `glp_erase_prob`.

The parameter `G` is the graph (network) program object, which specifies the maximum flow problem instance.

The parameter `names` is a flag. If it is `GLP_ON`, the routine uses symbolic names of the graph object components to assign symbolic names to the LP problem object components. If the flag is `GLP_OFF`, no symbolic names are assigned.

The parameter `s` specifies the ordinal number of the source node.

The parameter `t` specifies the ordinal number of the sink node.

The parameter `a_cap` specifies an offset of the field of type `double` in the arc data block, which contains u_{ij}, the upper bound to the arc flow (the arc capacity). If the upper bound is specified as `DBL_MAX`, it is assumed that $u_{ij} = \infty$, i.e. the arc is uncapacitated. If `a_cap` < 0, it is assumed that $u_{ij} = 1$ for all arcs.

Example

The example program below reads the maximum flow problem in DIMACS format from file '`sample.max`', converts the instance to LP, and then writes the resultant LP in CPLEX format to file '`maxflow.lp`'.

```
#include <stddef.h>
#include <glpk.h>

int main(void)
{     glp_graph *G;
      glp_prob *lp;
      int s, t;
      G = glp_create_graph(0, sizeof(double));
      glp_read_maxflow(G, &s, &t, 0, "sample.max");
      lp = glp_create_prob();
      glp_maxflow_lp(lp, G, GLP_ON, s, t, 0);
      glp_delete_graph(G);
      glp_write_lp(lp, NULL, "maxflow.lp");
      glp_delete_prob(lp);
      return 0;
}
```

If '`sample.max`' is the example data file from the previous subsection, the output '`maxflow.lp`' may look like follows:

```
Maximize
 obj: + x(1,4) + x(1,2)

Subject To
 r_1: + x(1,2) + x(1,4) >= 0
 r_2: - x(5,2) + x(2,3) + x(2,4) - x(1,2) = 0
 r_3: + x(3,5) + x(3,8) - x(2,3) = 0
 r_4: + x(4,5) - x(2,4) - x(1,4) = 0
 r_5: + x(5,2) + x(5,6) + x(5,7) - x(4,5) - x(3,5) = 0
 r_6: + x(6,7) + x(6,8) - x(5,6) = 0
 r_7: + x(7,9) - x(6,7) - x(5,7) = 0
 r_8: + x(8,9) - x(6,8) - x(3,8) = 0
 r_9: - x(8,9) - x(7,9) <= 0

Bounds
 0 <= x(1,4) <= 23
 0 <= x(1,2) <= 14
 0 <= x(2,4) <= 9
 0 <= x(2,3) <= 10
 0 <= x(3,8) <= 18
 0 <= x(3,5) <= 12
 0 <= x(4,5) <= 26
```

```
 0 <= x(5,7) <= 4
 0 <= x(5,6) <= 25
 0 <= x(5,2) <= 11
 0 <= x(6,8) <= 8
 0 <= x(6,7) <= 7
 0 <= x(7,9) <= 15
 0 <= x(8,9) <= 20

End
```

6.7.5 glp_maxflow_ffalg—solve maximum flow problem with Ford-Fulkerson algorithm

Synopsis

```
int glp_maxflow_ffalg(glp_graph *G, int s, int t, int a_cap,
      double *sol, int a_x, int v_cut);
```

Description

The routine `glp_mincost_ffalg` finds optimal solution to the maximum flow problem with the Ford-Fulkerson algorithm.[10] Note that this routine requires all the problem data to be integer-valued.

The parameter `G` is a graph (network) program object which specifies the maximum flow problem instance to be solved.

The parameter s specifies the ordinal number of the source node.

The parameter t specifies the ordinal number of the sink node.

The parameter `a_cap` specifies an offset of the field of type `double` in the arc data block, which contains u_{ij}, the upper bound to the arc flow (the arc capacity). This bound must be integer in the range [0, `INT_MAX`]. If `a_cap` < 0, it is assumed that $u_{ij} = 1$ for all arcs.

The parameter `sol` specifies a location, to which the routine stores the objective value (that is, the total flow from s to t) found. If `sol` is NULL, the objective value is not stored.

The parameter `a_x` specifies an offset of the field of type `double` in the arc data block, to which the routine stores x_{ij}, the arc flow found. If `a_x` < 0, the arc flow values are not stored.

The parameter `v_cut` specifies an offset of the field of type `int` in the vertex data block, to which the routine stores node flags corresponding to

[10] GLPK implementation of the Ford-Fulkerson algorithm is based on the following book: L. R. Ford, Jr., and D. R. Fulkerson, "Flows in Networks," The RAND Corp., Report R-375-PR (August 1962), Chap. I "Static Maximal Flow," pp. 30-33.

the optimal solution found: if the node flag is 1, the node is labelled, and if the node flag is 0, the node is unlabelled. The calling program may use these node flags to determine the *minimal cut*, which is a subset of arcs whose one endpoint is labelled and other is not. If `v_cut` < 0, the node flags are not stored.

Note that all solution components (the objective value and arc flows) computed by the routine are always integer-valued.

Returns

0	Optimal solution found.
`GLP_EDATA`	Unable to start the search, because some problem data are either not integer-valued or out of range.

Example

The example program shown below reads the maximum flow problem instance in DIMACS format from file '`sample.max`', solves it using the routine `glp_maxflow_ffalg`, and write the solution found to the standard output.

```
#include <stddef.h>
#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

typedef struct { int cut; } v_data;
typedef struct { double cap, x; } a_data;

#define node(v) ((v_data *)((v)->data))
#define arc(a)  ((a_data *)((a)->data))

int main(void)
{     glp_graph *G;
      glp_vertex *v, *w;
      glp_arc *a;
      int i, s, t, ret;
      double sol;
      G = glp_create_graph(sizeof(v_data), sizeof(a_data));
      glp_read_maxflow(G, &s, &t, offsetof(a_data, cap),
         "sample.max");
      ret = glp_maxflow_ffalg(G, s, t, offsetof(a_data, cap),
         &sol, offsetof(a_data, x), offsetof(v_data, cut));
      printf("ret = %d; sol = %5g\n", ret, sol);
      for (i = 1; i <= G->nv; i++)
      {  v = G->v[i];
         for (a = v->out; a != NULL; a = a->t_next)
```

```
         {  w = a->head;
            printf("x[%d->%d] = %5g (%d)\n", v->i, w->i,
               arc(a)->x, node(v)->cut ^ node(w)->cut);
         }
      }
      glp_delete_graph(G);
      return 0;
}
```

If '`sample.max`' is the example data file from the subsection describing the routine `glp_read_maxflow`, the output may look like follows:

```
Reading maximum flow problem data from 'sample.max'...
Flow network has 9 nodes and 14 arcs
24 lines were read
ret = 0; sol =    29
x[1->4] =    19 (0)
x[1->2] =    10 (0)
x[2->4] =     0 (0)
x[2->3] =    10 (1)
x[3->8] =    10 (0)
x[3->5] =     0 (1)
x[4->5] =    19 (0)
x[5->7] =     4 (1)
x[5->6] =    15 (0)
x[5->2] =     0 (0)
x[6->8] =     8 (1)
x[6->7] =     7 (1)
x[7->9] =    11 (0)
x[8->9] =    18 (0)
```

6.7.6 glp_rmfgen—Goldfarb's maximum flow problem generator

Synopsis

```
int glp_rmfgen(glp_graph *G, int *s, int *t, int a_cap,
      const int parm[1+5]);
```

Description

The routine `glp_rmfgen` is a GLPK version of the maximum flow problem generator developed by D. Goldfarb and M. Grigoriadis.[11,12,13]

The parameter `G` specifies the graph object, to which the generated problem data have to be stored. Note that on entry the graph object is erased with the routine `glp_erase_graph`.

The pointers `s` and `t` specify locations, to which the routine stores the source and sink node numbers, respectively. If `s` or `t` is `NULL`, corresponding node number is not stored.

The parameter `a_cap` specifies an offset of the field of type `double` in the arc data block, to which the routine stores the arc capacity. If `a_cap` < 0, the capacity is not stored.

The array `parm` contains description of the network to be generated:

`parm[0]`		not used
`parm[1]`	`seed`	random number seed (a positive integer)
`parm[2]`	`a`	frame size
`parm[3]`	`b`	depth
`parm[4]`	`c1`	minimal arc capacity
`parm[5]`	`c2`	maximal arc capacity

Returns

If the instance was successfully generated, the routine `glp_netgen` returns zero; otherwise, if specified parameters are inconsistent, the routine returns a non-zero error code.

[11] D. Goldfarb and M. D. Grigoriadis, "A computational comparison of the Dinic and network simplex methods for maximum flow." Annals of Op. Res. 13 (1988), pp. 83-123.

[12] U. Derigs and W. Meier, "Implementing Goldberg's max-flow algorithm: A computational investigation." Zeitschrift für Operations Research 33 (1989), pp. 383-403.

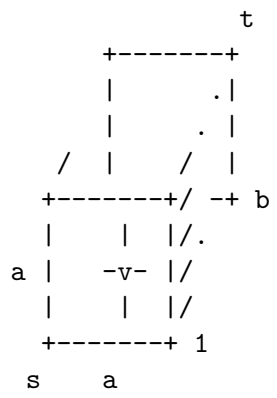
[13] The original code of RMFGEN implemented by Tamas Badics is publically available from `<ftp://dimacs.rutgers.edu/pub/netflow/generators/network/genrmf>`.

Comments[14]

The generated network is as follows. It has b pieces of frames of size $a \times a$. (So alltogether the number of vertices is $a \times a \times b$.)

In each frame all the vertices are connected with their neighbours (forth and back). In addition the vertices of a frame are connected one to one with the vertices of next frame using a random permutation of those vertices.

The source is the lower left vertex of the first frame, the sink is the upper right vertex of the b-th frame.



The capacities are randomly chosen integers from the range of $[c_1, c_2]$ in the case of interconnecting edges, and $c_2 \cdot a^2$ for the in-frame edges.

[14]This material is based on comments to the original version of RMFGEN.

6.8 Assignment problem

6.8.1 Background

Let there be given an undirected bipartite graph $G = (R \cup S, E)$, where R and S are disjoint sets of vertices (nodes), and $E \subseteq R \times S$ is a set of edges. Let also for each edge $e = (i, j) \in E$ there be given its cost c_{ij}. A *matching* (which in case of bipartite graph is also called *assignment*) $M \subseteq E$ in G is a set of pairwise non-adjacent edges, that is, no two edges in M share a common vertex. A matching, which matches all vertices of the graph, is called a *perfect matching*. Obviously, a perfect matching in bipartite graph $G = (R \cup S, E)$ defines some bijection $R \leftrightarrow S$.

The *assignment problem* has two different variants. In the first variant the problem is to find matching (assignment) M, which maximizes the sum:

$$\sum_{(i,j)\in M} c_{ij} \tag{10}$$

(so this variant is also called the *maximum weighted bipartite matching problem* or, if all $c_{ij} = 1$, the *maximum cardinality bipartite matching problem*). In the second, classic variant the problem is to find *perfect* matching (assignment) M, which minimizes or maximizes the sum (10).

An example of the assignment problem, which is the maximum weighted bipartite matching problem, is shown on Fig. 3.

The maximum weighted bipartite matching problem can be naturally formulated as the following LP problem:

maximize

$$z = \sum_{(i,j)\in E} c_{ij} x_{ij} \tag{11}$$

subject to

$$\sum_{(i,j)\in E} x_{ij} \le 1 \quad \text{for all } i \in R \tag{12}$$

$$\sum_{(i,j)\in E} x_{ij} \le 1 \quad \text{for all } j \in S \tag{13}$$

$$0 \le x_{ij} \le 1 \quad \text{for all } (i,j) \in E \tag{14}$$

where $x_{ij} = 1$ means that $(i, j) \in M$, and $x_{ij} = 0$ means that $(i, j) \notin M$.[15]

[15] The constraint matrix of LP formulation (11)—(14) is totally unimodular, due to which $x_{ij} \in \{0, 1\}$ for any basic solution.

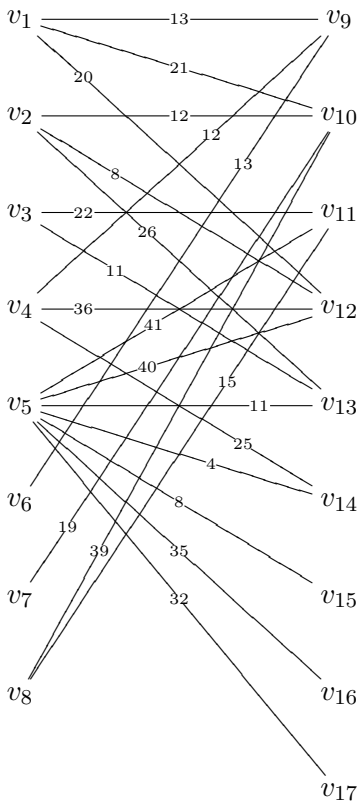


Fig. 3. An example of the assignment problem.

Similarly, the perfect assignment problem can be naturally formulated as the following LP problem:

minimize (or maximize)

$$z = \sum_{(i,j)\in E} c_{ij} x_{ij} \tag{15}$$

subject to

$$\sum_{(i,j)\in E} x_{ij} = 1 \quad \text{for all } i \in R \tag{16}$$

$$\sum_{(i,j)\in E} x_{ij} = 1 \quad \text{for all } j \in S \tag{17}$$

$$0 \le x_{ij} \le 1 \quad \text{for all } (i,j) \in E \tag{18}$$

where variables x_{ij} have the same meaning as for (11)—(14) above.

In GLPK an undirected bipartite graph $G = (R \cup S, E)$ is represented as directed graph $\overline{G} = (V, A)$, where $V = R \cup S$ and $A = \{(i, j) : (i, j) \in E\}$, i.e. every edge $(i, j) \in E$ in G corresponds to arc $(i \rightarrow j) \in A$ in $\overline{G}$.

6.8.2 glp_read_asnprob—read assignment problem data in DIMACS format

Synopsis

```
int glp_read_asnprob(glp_graph *G, int v_set, int a_cost,
      const char *fname);
```

Description

The routine `glp_read_asnprob` reads the assignment problem data from a text file in DIMACS format.

The parameter `G` specifies the graph object, to which the problem data have to be stored. Note that before reading data the current content of the graph object is completely erased with the routine `glp_erase_graph`.

The parameter `v_set` specifies an offset of the field of type `int` in the vertex data block, to which the routine stores the node set indicator:

0 — the node is in set R;

1 — the node is in set S.

If `v_set` < 0, the node set indicator is not stored.

The parameter `a_cost` specifies an offset of the field of type `double` in the arc data block, to which the routine stores the edge cost c_{ij}. If `a_cost` < 0, the edge cost is not stored.

The character string `fname` specifies the name of a text file to be read in. (If the file name name ends with the suffix '`.gz`', the file is assumed to be compressed, in which case the routine decompresses it "on the fly".)

Returns

If the operation was successful, the routine returns zero. Otherwise, it prints an error message and returns non-zero.

Example

```
typedef struct
{     /* vertex data block */
      ...
      int set;
      ...
} v_data;

typedef struct
{     /* arc data block */
      ...
      double cost;
      ...
} a_data;

int main(void)
{     glp_graph *G;
      int ret;
      G = glp_create_graph(sizeof(v_data), sizeof(a_data));
      ret = glp_read_asnprob(G, offsetof(v_data, set),
         offsetof(a_data, cost), "sample.asn");
      if (ret != 0) goto ...
      ...
}
```

DIMACS assignment problem format[16]

The DIMACS input file is a plain ASCII text file. It contains *lines* of several types described below. A line is terminated with an end-of-line character. Fields in each line are separated by at least one blank space. Each line begins with a one-character designator to identify the line type.

Note that DIMACS requires all numerical quantities to be integers in the range $[-2^{31},\ 2^{31}-1]$ while GLPK allows the quantities to be floating-point numbers.

Comment lines. Comment lines give human-readable information about the file and are ignored by programs. Comment lines can appear anywhere in the file. Each comment line begins with a lower-case character `c`.

```
c This is a comment line
```

[16] This material is based on the paper "The First DIMACS International Algorithm Implementation Challenge: Problem Definitions and Specifications", which is publically available at `http://dimacs.rutgers.edu/Challenges/`.

Problem line. There is one problem line per data file. The problem line must appear before any node or arc descriptor lines. It has the following format:

```
p asn NODES EDGES
```

The lower-case character `p` signifies that this is a problem line. The three-character problem designator `asn` identifies the file as containing specification information for the assignment problem. The `NODES` field contains an integer value specifying the total number of nodes in the graph (i.e. in both sets R and S). The `EDGES` field contains an integer value specifying the number of edges in the graph.

Node descriptors. All node descriptor lines must appear before all edge descriptor lines. The node descriptor lines lists the nodes in set R only, and all other nodes are assumed to be in set S. There is one node descriptor line for each such node, with the following format:

```
n ID
```

The lower-case character `n` signifies that this is a node descriptor line. The `ID` field gives a node identification number, an integer between 1 and `NODES`.

Edge descriptors. There is one edge descriptor line for each edge in the graph. Edge descriptor lines are of the following format:

```
a SRC DST COST
```

The lower-case character `a` signifies that this is an edge descriptor line. For each edge (i, j), where $i \in R$ and $j \in S$, the `SRC` field gives the identification number of vertex i, and the `DST` field gives the identification number of vertex j. Identification numbers are integers between 1 and `NODES`. The `COST` field contains the cost of edge (i, j).

Example. Below here is an example of the data file in DIMACS format corresponding to the assignment problem shown on Fig 3.

```
c sample.asn
c
c This is an example of the assignment problem data
c in DIMACS format.
c
p asn 17 22
c
n 1
n 2
n 3
n 4
n 5
n 6
n 7
n 8
c
a 1  9 13
a 1 10 21
a 1 12 20
a 2 10 12
a 2 12  8
a 2 13 26
a 3 11 22
a 3 13 11
a 4  9 12
a 4 12 36
a 4 14 25
a 5 11 41
a 5 12 40
a 5 13 11
a 5 14  4
a 5 15  8
a 5 16 35
a 5 17 32
a 6  9 13
a 7 10 19
a 8 10 39
a 8 11 15
c
c eof
```

6.8.3 glp_write_asnprob—write assignment problem data in DIMACS format

Synopsis

```
int glp_write_asnprob(glp_graph *G, int v_set, int a_cost,
      const char *fname);
```

Description

The routine `glp_write_asnprob` writes the assignment problem data to a text file in DIMACS format.

The parameter `G` is the graph program object, which specifies the assignment problem instance.

The parameter `v_set` specifies an offset of the field of type `int` in the vertex data block, which contains the node set indicator:

0 — the node is in set R;

1 — the node is in set S.

If `v_set` < 0, it is assumed that a node having no incoming arcs is in set R, and a node having no outgoing arcs is in set S.

The parameter `a_cost` specifies an offset of the field of type `double` in the arc data block, which contains c_{ij}, the edge cost. If `a_cost` < 0, it is assumed that $c_{ij} = 1$ for all edges.

The character string `fname` specifies a name of the text file to be written out. (If the file name ends with suffix '`.gz`', the file is assumed to be compressed, in which case the routine performs automatic compression on writing it.)

Note

The routine `glp_write_asnprob` does not check that the specified graph object correctly represents a bipartite graph. To make sure that the problem data are correct, use the routine `glp_check_asnprob`.

Returns

If the operation was successful, the routine returns zero. Otherwise, it prints an error message and returns non-zero.

6.8.4 glp_check_asnprob—check correctness of assignment problem data

Synopsis

```
int glp_check_asnprob(glp_graph *G, int v_set);
```

Description

The routine `glp_check_asnprob` checks that the specified graph object `G` correctly represents a bipartite graph.

The parameter `v_set` specifies an offset of the field of type `int` in the vertex data block, which contains the node set indicator:

0 — the node is in set R;

1 — the node is in set S.

If `v_set` < 0, it is assumed that a node having no incoming arcs is in set R, and a node having no outgoing arcs is in set S.

Returns

The routine `glp_check_asnprob` may return the following codes:

0 — the data are correct;

1 — the set indicator of some node is 0, however, that node has one or more incoming arcs;

2 — the set indicator of some node is 1, however, that node has one or more outgoing arcs;

3 — the set indicator of some node is invalid (neither 0 nor 1);

4 — some node has both incoming and outgoing arcs.

6.8.5 glp_asnprob_lp—convert assignment problem to LP

Synopsis

```
int glp_asnprob_lp(glp_prob *P, int form, glp_graph *G,
      int names, int v_set, int a_cost);
```

Description

The routine `glp_asnprob_lp` builds LP problem, which corresponds to the specified assignment problem.

The parameter `lp` is the resultant LP problem object to be built. Note that on entry its current content is erased with the routine `glp_erase_prob`.

The parameter `form` defines which LP formulation should be used:
`GLP_ASN_MIN` — perfect matching (15)—(18), minimization;
`GLP_ASN_MAX` — perfect matching (15)—(18), maximization;
`GLP_ASN_MMP` — maximum weighted matching (11)—(14).

The parameter `G` is the graph program object, which specifies the assignment problem instance.

The parameter `names` is a flag. If it is `GLP_ON`, the routine uses symbolic names of the graph object components to assign symbolic names to the LP problem object components. If the `flag` is `GLP_OFF`, no symbolic names are assigned.

The parameter `v_set` specifies an offset of the field of type `int` in the vertex data block, which contains the node set indicator:
0 — the node is in set R;
1 — the node is in set S.
If `v_set` < 0, it is assumed that a node having no incoming arcs is in set R, and a node having no outgoing arcs is in set S.

The parameter `a_cost` specifies an offset of the field of type `double` in the arc data block, which contains c_{ij}, the edge cost. If `a_cost` < 0, it is assumed that $c_{ij} = 1$ for all edges.

Returns

If the LP problem has been successfully built, the routine `glp_asnprob_lp` returns zero, otherwise, non-zero (see the routine `glp_check_asnprob`).

Example

The example program below reads the assignment problem instance in DIMACS format from file '`sample.asn`', converts the instance to LP (11)—(14), and writes the resultant LP in CPLEX format to file '`matching.lp`'.

```
#include <stddef.h>
#include <glpk.h>

typedef struct { int set; } v_data;
typedef struct { double cost; } a_data;

int main(void)
{     glp_graph *G;
      glp_prob *P;
      G = glp_create_graph(sizeof(v_data), sizeof(a_data));
      glp_read_asnprob(G, offsetof(v_data, set),
         offsetof(a_data, cost), "sample.asn");
```

```
      P = glp_create_prob();
      glp_asnprob_lp(P, GLP_ASN_MMP, G, GLP_ON,
         offsetof(v_data, set), offsetof(a_data, cost));
      glp_delete_graph(G);
      glp_write_lp(P, NULL, "matching.lp");
      glp_delete_prob(P);
      return 0;
}
```

If '`sample.asn`' is the example data file from the subsection describing the routine `glp_read_asnprob`, file '`matching.lp`' may look like follows:

```
Maximize
 obj: + 20 x(1,12) + 21 x(1,10) + 13 x(1,9) + 26 x(2,13) + 8 x(2,12)
 + 12 x(2,10) + 11 x(3,13) + 22 x(3,11) + 25 x(4,14) + 36 x(4,12)
 + 12 x(4,9) + 32 x(5,17) + 35 x(5,16) + 8 x(5,15) + 4 x(5,14)
 + 11 x(5,13) + 40 x(5,12) + 41 x(5,11) + 13 x(6,9) + 19 x(7,10)
 + 15 x(8,11) + 39 x(8,10)

Subject To
 r_1: + x(1,9) + x(1,10) + x(1,12) <= 1
 r_2: + x(2,10) + x(2,12) + x(2,13) <= 1
 r_3: + x(3,11) + x(3,13) <= 1
 r_4: + x(4,9) + x(4,12) + x(4,14) <= 1
 r_5: + x(5,11) + x(5,12) + x(5,13) + x(5,14) + x(5,15) + x(5,16)
 + x(5,17) <= 1
 r_6: + x(6,9) <= 1
 r_7: + x(7,10) <= 1
 r_8: + x(8,10) + x(8,11) <= 1
 r_9: + x(6,9) + x(4,9) + x(1,9) <= 1
 r_10: + x(8,10) + x(7,10) + x(2,10) + x(1,10) <= 1
 r_11: + x(8,11) + x(5,11) + x(3,11) <= 1
 r_12: + x(5,12) + x(4,12) + x(2,12) + x(1,12) <= 1
 r_13: + x(5,13) + x(3,13) + x(2,13) <= 1
 r_14: + x(5,14) + x(4,14) <= 1
 r_15: + x(5,15) <= 1
 r_16: + x(5,16) <= 1
 r_17: + x(5,17) <= 1

Bounds
 0 <= x(1,12) <= 1
 0 <= x(1,10) <= 1
 0 <= x(1,9) <= 1
 0 <= x(2,13) <= 1
 0 <= x(2,12) <= 1
 0 <= x(2,10) <= 1
 0 <= x(3,13) <= 1
 0 <= x(3,11) <= 1
 0 <= x(4,14) <= 1
```

```
 0 <= x(4,12) <= 1
 0 <= x(4,9) <= 1
 0 <= x(5,17) <= 1
 0 <= x(5,16) <= 1
 0 <= x(5,15) <= 1
 0 <= x(5,14) <= 1
 0 <= x(5,13) <= 1
 0 <= x(5,12) <= 1
 0 <= x(5,11) <= 1
 0 <= x(6,9) <= 1
 0 <= x(7,10) <= 1
 0 <= x(8,11) <= 1
 0 <= x(8,10) <= 1

End
```

6.8.6 glp_asnprob_okalg—solve assignment problem with out-of-kilter algorithm

Synopsis

```
int glp_asnprob_okalg(int form, glp_graph *G, int v_set,
      int a_cost, double *sol, int a_x);
```

Description

The routine `glp_mincost_okalg` finds optimal solution to the assignment problem with the out-of-kilter algorithm.[17] Note that this routine requires all the problem data to be integer-valued.

The parameter `form` defines which LP formulation should be used:

`GLP_ASN_MIN` — perfect matching (15)—(18), minimization;

`GLP_ASN_MAX` — perfect matching (15)—(18), maximization;

`GLP_ASN_MMP` — maximum weighted matching (11)—(14).

The parameter `G` is the graph program object, which specifies the assignment problem instance.

The parameter `v_set` specifies an offset of the field of type `int` in the vertex data block, which contains the node set indicator:

0 — the node is in set R;

1 — the node is in set S.

[17] GLPK implementation of the out-of-kilter algorithm is based on the following book: L. R. Ford, Jr., and D. R. Fulkerson, "Flows in Networks," The RAND Corp., Report R-375-PR (August 1962), Chap. III "Minimal Cost Flow Problems," pp. 113-26.

If `v_set` < 0, it is assumed that a node having no incoming arcs is in set R, and a node having no outgoing arcs is in set S.

The parameter `a_cost` specifies an offset of the field of type `double` in the arc data block, which contains c_{ij}, the edge cost. This value must be integer in the range [`-INT_MAX`, `+INT_MAX`]. If `a_cost` < 0, it is assumed that $c_{ij} = 1$ for all edges.

The parameter `sol` specifies a location, to which the routine stores the objective value (that is, the total cost) found. If `sol` is `NULL`, the objective value is not stored.

The parameter `a_x` specifies an offset of the field of type `int` in the arc data block, to which the routine stores x_{ij}. If `a_x` < 0, this value is not stored.

Returns

0	Optimal solution found.
`GLP_ENOPFS`	No (primal) feasible solution exists.
`GLP_EDATA`	Unable to start the search, because the assignment problem data are either incorrect (this error is detected by the routine `glp_check_asnprob`), not integer-valued or out of range.
`GLP_ERANGE`	The search was prematurely terminated because of integer overflow.
`GLP_EFAIL`	An error has been detected in the program logic. (If this code is returned for your problem instance, please report to `<bug-glpk@gnu.org>`.)

Comments

Since the out-of-kilter algorithm is designed to find a minimal cost circulation, the routine `glp_asnprob_okalg` converts the original graph to a network suitable for this algorithm in the following way:[18]

1) it replaces each edge (i, j) by arc $(i \rightarrow j)$, flow x_{ij} through which has zero lower bound ($l_{ij} = 0$), unity upper bound ($u_{ij} = 1$), and per-unit cost $+c_{ij}$ (in case of `GLP_ASN_MIN`), or $-c_{ij}$ (in case of `GLP_ASN_MAX` and `GLP_ASN_MMP`);

2) then it adds one auxiliary feedback node k;

[18] The conversion is performed internally and does not change the original graph program object passed to the routine.

3) for each original node $i \in R$ the routine adds auxiliary supply arc $(k \rightarrow i)$, flow x_{ki} through which is costless ($c_{ki} = 0$) and either fixed at 1 ($l_{ki} = u_{ki} = 1$, in case of `GLP_ASN_MIN` and `GLP_ASN_MAX`) or has zero lower bound and unity upper bound ($l_{ij} = 0$, $u_{ij} = 1$, in case of `GLP_ASN_MMP`);

4) similarly, for each original node $j \in S$ the routine adds auxiliary demand arc $(j \rightarrow k)$, flow x_{jk} through which is costless ($c_{jk} = 0$) and either fixed at 1 ($l_{jk} = u_{jk} = 1$, in case of `GLP_ASN_MIN` and `GLP_ASN_MAX`) or has zero lower bound and unity upper bound ($l_{jk} = 0$, $u_{jk} = 1$, in case of `GLP_ASN_MMP`).

Example

The example program shown below reads the assignment problem instance in DIMACS format from file '`sample.asn`', solves it by using the routine `glp_asnprob_okalg`, and writes the solution found to the standard output.

```
#include <stddef.h>
#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

typedef struct { int set; } v_data;
typedef struct { double cost; int x; } e_data;

#define node(v) ((v_data *)((v)->data))
#define edge(e) ((e_data *)((e)->data))

int main(void)
{     glp_graph *G;
      glp_vertex *v;
      glp_arc *e;
      int i, ret;
      double sol;
      G = glp_create_graph(sizeof(v_data), sizeof(e_data));
      glp_read_asnprob(G, offsetof(v_data, set),
         offsetof(e_data, cost), "sample.asn");
      ret = glp_asnprob_okalg(GLP_ASN_MMP, G,
         offsetof(v_data, set), offsetof(e_data, cost), &sol,
         offsetof(e_data, x));
      printf("ret = %d; sol = %5g\n", ret, sol);
      for (i = 1; i <= G->nv; i++)
      {  v = G->v[i];
         for (e = v->out; e != NULL; e = e->t_next)
            printf("edge %2d %2d: x = %d; c = %g\n",
               e->tail->i, e->head->i, edge(e)->x, edge(e)->cost);
      }
```

```
      glp_delete_graph(G);
      return 0;
}
```

If '`sample.asn`' is the example data file from the subsection describing the routine `glp_read_asnprob`, the output may look like follows:

```
Reading assignment problem data from 'sample.asn'...
Assignment problem has 8 + 9 = 17 nodes and 22 arcs
38 lines were read
ret = 0; sol =   180
edge  1 12: x = 1; c = 20
edge  1 10: x = 0; c = 21
edge  1  9: x = 0; c = 13
edge  2 13: x = 1; c = 26
edge  2 12: x = 0; c = 8
edge  2 10: x = 0; c = 12
edge  3 13: x = 0; c = 11
edge  3 11: x = 1; c = 22
edge  4 14: x = 1; c = 25
edge  4 12: x = 0; c = 36
edge  4  9: x = 0; c = 12
edge  5 17: x = 0; c = 32
edge  5 16: x = 1; c = 35
edge  5 15: x = 0; c = 8
edge  5 14: x = 0; c = 4
edge  5 13: x = 0; c = 11
edge  5 12: x = 0; c = 40
edge  5 11: x = 0; c = 41
edge  6  9: x = 1; c = 13
edge  7 10: x = 0; c = 19
edge  8 11: x = 0; c = 15
edge  8 10: x = 1; c = 39
```

6.8.7 glp_asnprob_hall—find bipartite matching of maximum cardinality

Synopsis

```
int glp_asnprob_hall(glp_graph *G, int v_set, int a_x);
```

Description

The routine `glp_asnprob_hall` finds a matching of maximal cardinality in the specified bipartite graph. It uses a version of the Fortran routine `MC21A` developed by I. S. Duff[19], which implements Hall's algorithm.[20]

The parameter `G` is a pointer to the graph program object.

The parameter `v_set` specifies an offset of the field of type `int` in the vertex data block, which contains the node set indicator:

0 — the node is in set R;

1 — the node is in set S.

If `v_set` < 0, it is assumed that a node having no incoming arcs is in set R, and a node having no outgoing arcs is in set S.

The parameter `a_x` specifies an offset of the field of type `int` in the arc data block, to which the routine stores x_{ij}. If `a_x` < 0, this value is not stored.

Returns

The routine `glp_asnprob_hall` returns the cardinality of the matching found. However, if the specified graph is incorrect (as detected by the routine `glp_check_asnprob`), this routine returns a negative value.

Comments

The same solution may be obtained with the routine `glp_asnprob_okalg` (for LP formulation `GLP_ASN_MMP` and all edge costs equal to 1). However, the routine `glp_asnprob_hall` is much faster.

[19] I. S. Duff, Algorithm 575: Permutations for zero-free diagonal, ACM Trans. on Math. Softw. 7 (1981), pp. 387-390.

[20] M. Hall, "An Algorithm for Distinct Representatives," Am. Math. Monthly 63 (1956), pp. 716-717.

Example

The example program shown below reads the assignment problem instance in DIMACS format from file '`sample.asn`', finds a bipartite matching of maximal cardinality by using the routine `glp_asnprob_hall`, and writes the solution found to the standard output.

```
#include <stddef.h>
#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

typedef struct { int set; } v_data;
typedef struct { int x;   } e_data;

#define node(v) ((v_data *)((v)->data))
#define edge(e) ((e_data *)((e)->data))

int main(void)
{     glp_graph *G;
      glp_vertex *v;
      glp_arc *e;
      int i, card;
      G = glp_create_graph(sizeof(v_data), sizeof(e_data));
      glp_read_asnprob(G, offsetof(v_data, set), -1,
         "sample.asn");
      card = glp_asnprob_hall(G, offsetof(v_data, set),
         offsetof(e_data, x));
      printf("card = %d\n", card);
      for (i = 1; i <= G->nv; i++)
      {  v = G->v[i];
         for (e = v->out; e != NULL; e = e->t_next)
            printf("edge %2d %2d: x = %d\n",
               e->tail->i, e->head->i, edge(e)->x);
      }
      glp_delete_graph(G);
      return 0;
}
```

If '`sample.asn`' is the example data file from the subsection describing the routine `glp_read_asnprob`, the output may look like follows:

```
Reading assignment problem data from 'sample.asn'...
Assignment problem has 8 + 9 = 17 nodes and 22 arcs
38 lines were read
card = 7
edge  1 12: x = 1
edge  1 10: x = 0
```

```
edge  1  9: x = 0
edge  2 13: x = 1
edge  2 12: x = 0
edge  2 10: x = 0
edge  3 13: x = 0
edge  3 11: x = 1
edge  4 14: x = 1
edge  4 12: x = 0
edge  4  9: x = 0
edge  5 17: x = 1
edge  5 16: x = 0
edge  5 15: x = 0
edge  5 14: x = 0
edge  5 13: x = 0
edge  5 12: x = 0
edge  5 11: x = 0
edge  6  9: x = 1
edge  7 10: x = 1
edge  8 11: x = 0
edge  8 10: x = 0
```

6.9 Maximum clique problem

6.9.1 Background

The *Maximum Clique Problem (MCP)* is a classic combinatorial optimization problem. Given an undirected graph $G = (V, E)$, where V is a set of vertices, and E is a set of edges, this problem is to find the largest *clique* $C \subseteq G$, i.e. the largest induced complete subgraph. A generalization of this problem is the *Maximum Weight Clique Problem (MWCP)*, which is to find a clique $C \subseteq G$ of the largest weight $\sum_{v \in C} w(v) \to \max$, where $w(v)$ is a weight of vertex $v \in V$.

An example of the Maximum Weight Clique Problem is shown on Fig. 4.

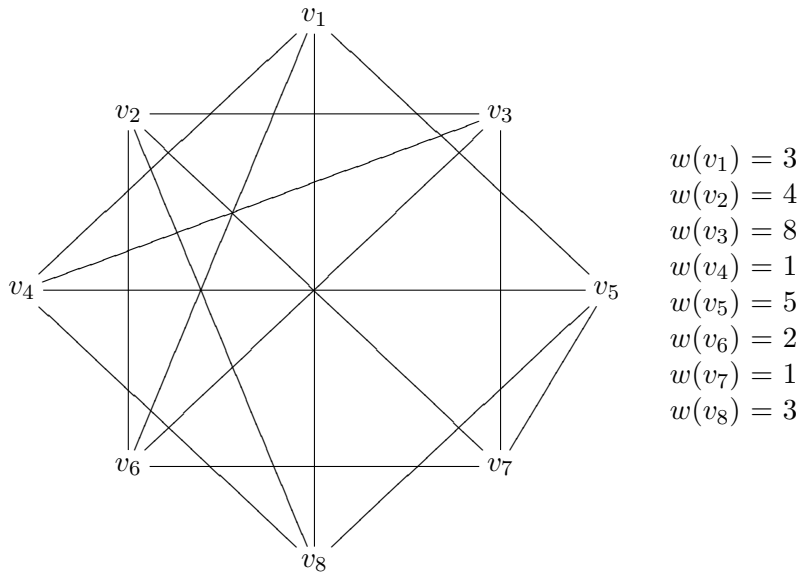


Fig. 4. An example of the Maximum Weight Clique Problem.

6.9.2 glp_wclique_exact—find maximum weight clique with exact algorithm

Synopsis

```
int glp_wclique_exact(glp_graph *G, int v_wgt, double *sol,
      int v_set);
```

Description

The routine *glp_wclique_exact* finds a maximum weight clique in the specified undirected graph with the exact algorithm developed by Patric Östergård.[21]

The parameter G is the program object, which specifies an undirected graph. Each arc $(x \rightarrow y)$ in G is considered as edge (x, y), self-loops are ignored, and multiple edges, if present, are replaced (internally) by simple edges.

The parameter *v_wgt* specifies an offset of the field of type **double** in the vertex data block, which contains a weight of corresponding vertex. Vertex weights must be integer-valued in the range [0, *INT_MAX*]. If $v\_wgt < 0$, it is assumed that all vertices of the graph have the weight 1.

The parameter *sol* specifies a location, to which the routine stores the weight of the clique found (the clique weight is the sum of weights of all vertices included in the clique.) If *sol* is *NULL*, the solution is not stored.

The parameter *v_set* specifies an offset of the field of type **int** in the vertex data block, to which the routines stores a vertex flag: 1 means that the corresponding vertex is included in the clique found, and 0 otherwise. If $v\_set < 0$, vertex flags are not stored.

Returns

0	Optimal solution found.
`GLP_EDATA`	Unable to start the search, because some vertex weights are either not integer-valued or out of range. This code is also returned if the sum of weights of all vertices exceeds *INT_MAX*.

[21] P. R. J. Östergård, A new algorithm for the maximum-weight clique problem, Nordic J. of Computing, Vol. 8, No. 4, 2001, pp. 424–36.

Notes

1. The routine *glp_wclique_exact* finds exact solution. Since both MCP and MWCP problems are NP-complete, the algorithm may require exponential time in worst cases.

2. Internally the specified graph is converted to an adjacency matrix in *dense* format. This requires about $|V|^2/16$ bytes of memory, where $|V|$ is the number of vertices in the graph.

Example

The example program shown below reads a MWCP instance in DIMACS clique/coloring format from file '`sample.clq`', finds the clique of largest weight, and writes the solution found to the standard output.

```
#include <stddef.h>
#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

typedef struct { double wgt; int set; } v_data;

#define vertex(v) ((v_data *)((v)->data))

int main(void)
{     glp_graph *G;
      glp_vertex *v;
      int i, ret;
      double sol;
      G = glp_create_graph(sizeof(v_data), 0);
      glp_read_ccdata(G, offsetof(v_data, wgt), "sample.clq");
      ret = glp_wclique_exact(G, offsetof(v_data, wgt), &sol,
         offsetof(v_data, set));
      printf("ret = %d; sol = %g\n", ret, sol);
      for (i = 1; i <= G->nv; i++)
      {  v = G->v[i];
         printf("vertex %d: weight = %g, flag = %d\n",
            i, vertex(v)->wgt, vertex(v)->set);
      }
      glp_delete_graph(G);
      return 0;
}
```

For the example shown on Fig. 4 the data file may look like follows:

```
c sample.clq
c
c This is an example of the Maximum Weight Clique
c Problem in DIMACS clique/coloring format.
c
p edge 8 16
n 1 3
n 2 4
n 3 8
n 5 5
n 6 2
n 8 3
e 1 4
e 1 5
e 1 6
e 1 8
e 2 3
e 2 6
e 2 7
e 2 8
e 3 4
e 3 6
e 3 7
e 4 5
e 4 8
e 5 7
e 5 8
e 6 7
c
c eof
```

The corresponding output from the example program is the following:

```
Reading graph from ‘sample.clq’...
Graph has 8 vertices and 16 edges
28 lines were read
ret = 0; sol = 15
vertex 1: weight = 3, flag = 0
vertex 2: weight = 4, flag = 1
vertex 3: weight = 8, flag = 1
vertex 4: weight = 1, flag = 0
vertex 5: weight = 5, flag = 0
vertex 6: weight = 2, flag = 1
vertex 7: weight = 1, flag = 1
vertex 8: weight = 3, flag = 0
```

Chapter 7

Miscellaneous API Routines

7.1 Library environment routines

7.1.1 glp_long—64-bit integer data type

Some GLPK API routines use 64-bit integer data type, which is declared in the header `glpk.h` as follows:

```
typedef struct { int lo, hi; } glp_long;
```

where `lo` contains low 32 bits, and `hi` contains high 32 bits of 64-bit integer value.[1]

7.1.2 glp_version—determine library version

Synopsis

```
const char *glp_version(void);
```

Returns

The routine `glp_version` returns a pointer to a null-terminated character string, which specifies the version of the GLPK library in the form `"X.Y"`, where '`X`' is the major version number, and '`Y`' is the minor version number, for example, `"4.16"`.

[1] GLPK conforms to ILP32, LLP64, and LP64 programming models, where the built-in type `int` corresponds to 32-bit integers.

Example

```
printf("GLPK version is %s\n", glp_version());
```

7.1.3 glp_printf—write formatted output to terminal

Synopsis

```
void glp_printf(const char *fmt, ...);
```

Description

The routine `glp_printf` uses the format control string `fmt` to format its parameters and writes the formatted output to the terminal.

This routine is a replacement of the standard C function `printf` and used by all GLPK routines to perform terminal output. The application program may use `glp_printf` for the same purpose that allows controlling its terminal output with the routines `glp_term_out` and `glp_term_hook`.

7.1.4 glp_vprintf—write formatted output to terminal

Synopsis

```
void glp_vprintf(const char *fmt, va_list arg);
```

Description

The routine `glp_vprintf` uses the format control string `fmt` to format its parameters specified by the list `arg` and writes the formatted output to the terminal.

This routine is a replacement of the standard C function `vprintf` and used by all GLPK routines to perform terminal output. The application program may use `glp_vprintf` for the same purpose that allows controlling its terminal output with the routines `glp_term_out` and `glp_term_hook`.

7.1.5 glp_term_out—enable/disable terminal output

Synopsis

```
int glp_term_out(int flag);
```

Description

Depending on the parameter flag the routine `glp_term_out` enables or disables terminal output performed by glpk routines:

`GLP_ON` — enable terminal output;
`GLP_OFF` — disable terminal output.

Returns

The routine `glp_term_out` returns the current terminal output mode before it was changed (`GLP_ON` or `GLP_OFF`).

7.1.6 glp_term_hook—intercept terminal output

Synopsis

```
void glp_term_hook(int (*func)(void *info, const char *s),
      void *info);
```

Description

The routine `glp_term_hook` installs the user-defined hook routine to intercept all terminal output performed by GLPK routines.

The parameter *func* specifies the user-defined hook routine. It is called from an internal printing routine, which passes to it two parameters: *info* and *s*. The parameter *info* is a transit pointer specified in corresponding call to the routine `glp_term_hook`; it may be used to pass some additional information to the hook routine. The parameter *s* is a pointer to the null terminated character string, which is intended to be written to the terminal. If the hook routine returns zero, the printing routine writes the string *s* to the terminal in a usual way; otherwise, if the hook routine returns non-zero, no terminal output is performed.

To uninstall the hook routine both parameters *func* and *info* should be specified as `NULL`.

Example

```
static int hook(void *info, const char *s)
{     FILE *foo = info;
      fputs(s, foo);
      return 1;
}

int main(void)
{     FILE *foo;
      . . .
      /* redirect terminal output */
      glp_term_hook(hook, foo);
      . . .
      /* resume terminal output */
      glp_term_hook(NULL, NULL);
      . . .
}
```

7.1.7 glp_malloc—allocate memory block

Synopsis

```
void *glp_malloc(int size);
```

Description

The routine `glp_malloc` dynamically allocates a memory block of `size` bytes long. Should note that:

1) the parameter `size` must be positive;

2) being allocated the memory block contains arbitrary data, that is, it is *not* initialized by binary zeros;

3) if the block cannot be allocated due to insufficient memory, the routine prints an error message and abnormally terminates the program.

This routine is a replacement of the standard C function `malloc` and used by all GLPK routines for dynamic memory allocation. The application program may use `glp_malloc` for the same purpose.

Returns

The routine `glp_malloc` returns a pointer to the memory block allocated. To free this block the routine `glp_free` (not the standard C function `free`!) must be used.

7.1.8 glp_calloc—allocate memory block

Synopsis

```
void *glp_calloc(int n, int size);
```

Description

The routine `glp_calloc` dynamically allocates a memory block of `n`×`size` bytes long. Should note that:

1) both parameters `n` and `size` must be positive;

2) being allocated the memory block contains arbitrary data, that is, it is *not* initialized by binary zeros;

3) if the block cannot be allocated due to insufficient memory, the routine prints an error message and abnormally terminates the program.

This routine is a replacement of the standard C function `calloc` (with exception that the block is not cleaned) and used by all GLPK routines for dynamic memory allocation. The application program may use `glp_calloc` for the same purpose.

Returns

The routine `glp_calloc` returns a pointer to the memory block allocated. To free this block the routine `glp_free` (not the standard C function `free`!) must be used.

7.1.9 glp_free—free memory block

Synopsis

```
void glp_free(void *ptr);
```

Description

The routine `glp_free` frees (deallocates) a memory block pointed to by `ptr`, which was previously allocated by the routine `glp_malloc` or `glp_calloc`. Note that the pointer `ptr` must valid and must not be `NULL`.

This routine is a replacement of the standard C function `free` and used by all GLPK routines for dynamic memory allocation. The application program may use `glp_free` for the same purpose.

7.1.10 glp_mem_usage—get memory usage information

Synopsis

```
void glp_mem_usage(int *count, int *cpeak, glp_long *total,
      glp_long *tpeak);
```

Description

The routine `glp_mem_usage` reports some information about utilization of the memory by the routines `glp_malloc`, `glp_calloc`, and `glp_free`. Information is stored to locations specified by corresponding parameters (see below). Any parameter can be specified as `NULL`, in which case corresponding information is not stored.

`*count` is the number of currently allocated memory blocks.

`*cpeak` is the peak value of `*count` reached since the initialization of the GLPK library environment.

`*total` is the total amount, in bytes, of currently allocated memory blocks.

`*tpeak` is the peak value of `*total` reached since the initialization of the GLPK library envirionment.

Example

```
glp_mem_usage(&count, NULL, NULL, NULL);
printf("%d memory block(s) are still allocated\n", count);
```

7.1.11 glp_mem_limit—set memory usage limit

Synopsis

```
void glp_mem_limit(int limit);
```

Description

The routine `glp_mem_limit` limits the amount of memory available for dynamic allocation (with the routines `glp_malloc` and `glp_calloc`) to `limit` megabytes.

7.1.12 glp_assert—check logical condition

Synopsis

```
void glp_assert(int expr);
```

Description

The routine `glp_assert` (implemented as a macro) checks a logical condition specified by the expression `expr`. If the condition is true (non-zero), the routine does nothing; otherwise, if the condition is false (zero), the routine prints an error message and abnormally terminates the program.

This routine is a replacement of the standard C function `assert` and used by all GLPK routines to check program logic. The application program may use `glp_assert` for the same purpose.

7.1.13 glp_free_env—free GLPK library environment

Synopsis

```
void glp_free_env(void);
```

Description

The routine `glp_free_env` frees all resources used by GLPK routines (memory blocks, etc.) which are currently still in use.

Usage notes

Normally the application program does not need to call this routine, because GLPK routines always free all unused resources. However, if the application program even has deleted all problem objects, there will be several memory blocks still allocated for the internal library needs. For some reasons the application program may want GLPK to free this memory, in which case it should call `glp_free_env`.

Note that a call to `glp_free_env` invalidates all problem objects which still exist.

7.2 Plain data file reading routines

7.2.1 Introduction

On developing simple applications to solve optimization problems it is often needed to read data from plain text files. To do this the standard C function `fscanf` may be used, however, it is not convenient; for example, if it scans an integer number according to the format specification '`%d`', and that number is coded incorrectly, no diagnostics is provided.

This section describes a set of GLPK API routines, which may be used in application programs to simplify reading data from plain text files.

Example 1

The following main program reads ten integer numbers from plain text file `data.txt` and prints their sum.

```
/* sdfsamp1.c */

#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

int main(void)
{     glp_data *data;
      int j, num, sum;
      /* open plain data file */
      data = glp_sdf_open_file("data.txt");
      if (data == NULL) exit(EXIT_FAILURE);
      sum = 0;
      for (j = 1; j <= 10; j++)
      {  /* read next integer number */
         num = glp_sdf_read_int(data);
         sum += num;
      }
      printf("sum = %d\n", sum);
      /* close plain data file */
      glp_sdf_close_file(data);
      return 0;
}

/* eof */
```

The input data are coded in free format. For example, the file `data.txt` may look like this:

```
123 65 432 890 -12 743 895 -7 111 326
```

or like this:

```
123   65  432  890  -12
743  895   -7  111  326
```

If the input data file contains incorrect data, the routine `glp_sdf_read_int` prints an error message and, if no error handling is provided by the application program, abnormally terminates program execution. For example, the file `data.txt` could contain the following data:

```
123   65  432  890  -12
743  895   =7  111  326
```

in which case the error message would be the following:

```
data.txt:2: cannot convert ‘=7’ to integer
```

Example 2

As it was said above, by default any attempt to read incorrect data leads to abnormal termination. However, sometimes it is desirable to catch such errors. This feature is illustrated by the following main program, which does the same job as in the previous example.

```
/* sdfsamp2.c */

#include <setjmp.h>
#include <stdio.h>
#include <stdlib.h>
#include <glpk.h>

int main(void)
{     glp_data *data;
      jmp_buf jump;
      int j, num, sum, ret;
      /* open plain data file */
      data = glp_sdf_open_file("data.txt");
      if (data == NULL)
      {  ret = EXIT_FAILURE;
         goto done;
      }
      /* set up error handling */
      if (setjmp(jump))
      {  ret = EXIT_FAILURE;
         goto done;
      }
```

```
      glp_sdf_set_jump(data, jump);
      /* read and process data */
      sum = 0;
      for (j = 1; j <= 10; j++)
      {  /* read next integer number */
         num = glp_sdf_read_int(data);
         if (abs(num) > 1000)
            glp_sdf_error(data, "integer %d too big\n", num);
         if (num < 0)
            glp_sdf_warning(data, "integer %d is negative\n", num);
         sum += num;
      }
      printf("sum = %d\n", sum);
      ret = EXIT_SUCCESS;
done: /* close plain data file */
      if (data != NULL) glp_sdf_close_file(data);
      return ret;
}

/* eof */
```

7.2.2 glp_sdf_open_file—open plain data file

Synopsis

```
glp_data *glp_sdf_open_file(const char *fname);
```

Description

The routine `glp_sdf_open_file` opens a plain data file, whose name is specified by the character string `fname`.

Returns

If the operation was successful, the routine `glp_sdf_open_file` returns a pointer to the opaque program object of the type `glp_data`[2] associated with the plain data file. Otherwise, if the operation failed, the routine prints an error message and returns `NULL`.

Note

The application program should use the pointer returned by the routine `glp_sdf_open_file` to perform all subsequent operations on the data file.

[2] This data structure is declared in the header file `glpk.h`.

7.2.3 glp_sdf_set_jump—set up error handling

Synopsis

```
void glp_sdf_set_jump(glp_data *data, jmp_buf jump);
```

Description

The routine `glp_sdf_set_jump` sets up error handling for the plain data file specified by the parameter `data`.

The parameter `jump` specifies the environment buffer, which must be initialized with the standard C function `setjmp` prior to call to the routine `glp_sdf_set_jump`. Detecting any incorrect data in the corresponding plain data file will cause non-local "go to" by a call to the standard C function `longjmp`.

The parameter `jump` can be specified as `NULL`, in which case the routine `glp_sdf_set_jump` restores the default behavior, in which case detecting incorrect data leads to abnormal termination.

7.2.4 glp_sdf_error—print error message

Synopsis

```
void glp_sdf_error(glp_data *data, const char *fmt, ...);
```

Description

The routine `glp_sdf_error` prints an error message related to the plain data file specified by the parameter `data`. If error handing was not previously provided, the routine then abnormally terminates execution of the application program. Otherwise, it signals about the error by a call to the standard C function `longjmp`.

The character string `fmt` and optional parameters following it have the same meaning as for the standard C function `printf`.

The message produced by the routine `glp_sdf_error` looks like follows:

file:*line*: *message text*

where *file* is the filename passed to the routine `glp_sdf_open` and *line* is the current line number.

7.2.5 glp_sdf_warning—print warning message

Synopsis

```
void glp_sdf_warning(glp_data *data, const char *fmt, ...);
```

Description

The routine `glp_sdf_warning` prints a warning message related to the plain data file specified by the parameter `data`.

The character string `fmt` and optional parameters following it have the same meaning as for the standard C function `printf`.

The message produced by the routine `glp_sdf_warning` looks like follows:

file:*line*: `warning:` *message text*

where *file* is the filename passed to the routine `glp_sdf_open` and *line* is the current line number.

7.2.6 glp_sdf_read_int—read integer number

Synopsis

```
int glp_sdf_read_int(glp_data *data);
```

Description

The routine `glp_sdf_read_int` skips optional white-space characters and then reads an integer number from the plain data file specified by the parameter `data`. If the operation failed, the routine `glp_sdf_read_int` calls the routine `glp_sdf_error` (see above).

Returns

The routine `glp_sdf_read_int` returns the integer number read.

7.2.7 glp_sdf_read_num—read floating-point number

Synopsis

```
double glp_sdf_read_num(glp_data *data);
```

Description

The routine `glp_sdf_read_num` skips optional white-space characters and then reads a floating-point number from the plain data file specified by the parameter `data`. If the operation failed, the routine `glp_sdf_num` calls the routine `glp_sdf_error` (see above).

Returns

The routine `glp_sdf_read_num` returns the floating-point number read.

7.2.8 glp_sdf_read_item—read data item

Synopsis

```
const char *glp_sdf_read_item(glp_data *data);
```

Description

The routine `glp_sdf_read_item` skips optional white-space characters and then reads a data item from the plain data file specified by the parameter `data`. If the operation failed, the routine `glp_sdf_read_item` calls the routine `glp_sdf_error` (see above).

Data item is a sequence of 1 to 255 arbitrary graphic characters delimited by white-space characters. Data items may be used to represent symbolic names, identifiers, etc.

Returns

The routine `glp_sdf_read_item` returns a pointer to the internal buffer, which contains the data item read in the form of a null-terminated character string.

7.2.9 glp_sdf_read_text—read text until end of line

Synopsis

```
const char *glp_sdf_read_text(glp_data *data);
```

Description

The routine `glp_sdf_read_text` reads a text from the plain data file specified by the parameter `data`.

Reading starts from the current position and extends until end of the current line. Initial and trailing white-space characters as well as the newline character are not included in the text.

Returns

The routine `glp_sdf_read_text` returns a pointer to the internal buffer, which contains the text read in the form of a null-terminated character string.

7.2.10 glp_sdf_line—determine current line number

Synopsis

```
int glp_sdf_line(glp_data *data);
```

Returns

The routine `glp_sdf_line` returns the current line number for the plain data file specified by the parameter `data`.

7.2.11 glp_sdf_close_file—close plain data file

Synopsis

```
void glp_sdf_close_file(glp_data *data);
```

Description

The routine `glp_sdf_close_file` closes the plain data file specified by the parameter `data` and frees all the resources allocated to this program object.

Appendix A

Installing GLPK on Your Computer

A.1 Downloading the distribution tarball

The distribution tarball of the most recent version of the GLPK package can be found on http://ftp.gnu.org/gnu/glpk/ [via http] and ftp://ftp.gnu.org/gnu/glpk/ [via FTP]. It can also be found on one of the FTP mirrors; see http://www.gnu.org/prep/ftp.html. Please use a mirror if possible.

To make sure that the GLPK distribution tarball you have downloaded is intact you need to download the corresponding '`.sig`' file and run a command like this:

```
gpg --verify glpk-4.38.tar.gz.sig
```

If that command fails because you do not have the required public key, run the following command to import it:

```
gpg --keyserver keys.gnupg.net --recv-keys 5981E818
```

and then re-run the previous command.

A.2 Unpacking the distribution tarball

The GLPK package (like all other GNU software) is distributed in the form of packed archive. This is one file named `glpk-X.Y.tar.gz`, where X is the major version number and Y is the minor version number.

In order to prepare the distribution for installation you should:

1. Copy the GLPK distribution file to a working subdirectory.

2. Unpack the distribution file with the following command:

```
gzip -d glpk-X.Y.tar.gz
```

that renames the distribution file to `glpk-X.Y.tar`.

3. Unarchive the distribution file with the following command:

```
tar -x < glpk-X.Y.tar
```

that automatically creates the subdirectory `glpk-X.Y` containing the GLPK distribution.

A.3 Configuring the package

After unpacking and unarchiving the GLPK distribution you should configure the package, i.e. automatically tune it for your platform.

Normally, you should just `cd` to the subdirectory `glpk-X.Y` and run the configure script, e.g.

```
./configure
```

The '`configure`' shell script attempts to guess correct values for various system-dependent variables used during compilation. It uses those values to create a '`Makefile`' in each directory of the package. It also creates file '`config.h`' containing platform-dependent definitions. Finally, it creates a shell script '`config.status`' that you can run in the future to recreate the current configuration, a file '`config.cache`' that saves the results of its tests to speed up reconfiguring, and a file '`config.log`' containing compiler output (useful mainly for debugging '`configure`').

Running '`configure`' takes about a few minutes. While it is running, it displays some informational messages that tell you what it is doing. If you don't want to see these messages, run '`configure`' with its standard output redirected to '`dev/null`'; for example, '`./configure > /dev/null`'.

By default both static and shared versions of the GLPK library will be compiled. Compilation of the shared librariy can be turned off by specifying the '`--disable-shared`' option to '`configure`', e.g.

```
./configure --disable-shared
```

If you encounter problems building the library try using the above option, because some platforms do not support shared libraries.

The GLPK package has some optional features listed below. By default all these features are disabled. To enable a feature the corresponding option should be passed to the configure script.

`--with-gmp` Enable using the GNU MP bignum library

This feature allows the exact simplex solver to use the GNU MP bignum library. If it is disabled, the exact simplex solver uses the GLPK bignum module, which provides the same functionality as GNU MP, however, it is much less efficient.

For details about the GNU MP bignum library see its web page at http://gmplib.org/.

`--with-zlib` Enable using the zlib data compression library

This feature allows GLPK API routines and the stand-alone solver to read and write compressed data files performing compression and decompression "on the fly" (compressed data files are recognized by suffix '`.gz`' in the file name). It may be useful in case of large MPS files to save the disk space.

For details about the zlib compression library see its web page at http://www.zlib.net/.

`--enable-dl` The same as '`--enable-dl=ltdl`'
`--enable-dl=ltdl` Enable shared library support (GNU)
`--enable-dl=dlfcn` Enable shared library support (POSIX)

Currently this feature is only needed to provide dynamic linking to ODBC and MySQL shared libraries (see below).

For details about the GNU shared library support see the manual at http://www.gnu.org/software/libtool/manual/.

`--enable-odbc` Enable using ODBC table driver (`libiodbc`)
`--enable-odbc=unix` Enable using ODBC table driver (`libodbc`)

This feature allows transmitting data between MathProg model objects and relational databases accessed through ODBC.

For more details about this feature see the supplement "Using Data Tables in the GNU MathProg Modeling Language" (`doc/tables.pdf`).

`--enable-mysql` Enable using MySQL table driver (`libmysql`)

This feature allows transmitting data between MathProg model objects and MySQL relational databases.

For more details about this feature see the supplement "Using Data Tables in the GNU MathProg Modeling Language" (`doc/tables.pdf`).

A.4 Compiling the package

Normally, you can compile (build) the package by typing the command:

```
make
```

It reads '`Makefile`' generated by '`configure`' and performs all necessary jobs.

If you want, you can override the '`make`' variables `CFLAGS` and `LDFLAGS` like this:

```
make CFLAGS=-O2 LDFLAGS=-s
```

To compile the package in a different directory from the one containing the source code, you must use a version of '`make`' that supports '`VPATH`' variable, such as GNU '`make`'. '`cd`' to the directory where you want the object files and executables to go and run the '`configure`' script. '`configure`' automatically checks for the source code in the directory that '`configure`' is in and in '`..`'. If for some reason '`configure`' is not in the source code directory that you are configuring, then it will report that it can't find the source code. In that case, run '`configure`' with the option '`--srcdir=DIR`', where `DIR` is the directory that contains the source code.

Some systems require unusual options for compilation or linking that the '`configure`' script does not know about. You can give '`configure`' initial values for variables by setting them in the environment. Using a Bourne-compatible shell, you can do that on the command line like this:

```
CC=c89 CFLAGS=-O2 LIBS=-lposix ./configure
```

Or on systems that have the '`env`' program, you can do it like this:

```
env CPPFLAGS=-I/usr/local/include LDFLAGS=-s ./configure
```

Here are the '`make`' variables that you might want to override with environment variables when running '`configure`'.

For these variables, any value given in the environment overrides the value that '`configure`' would choose:

`CC` C compiler program. The default is '`cc`'.

`INSTALL` Program used to install files. The default value is '`install`' if you have it, otherwise '`cp`'.

For these variables, any value given in the environment is added to the value that '`configure`' chooses:

`DEFS` Configuration options, in the form '`-Dfoo -Dbar` ...'.

`LIBS` Libraries to link with, in the form '`-lfoo -lbar` ...'.

A.5 Checking the package

To check the package, i.e. to run some tests included in the package, you can use the following command:

```
make check
```

A.6 Installing the package

Normally, to install the GLPK package you should type the following command:

```
make install
```

By default, '`make install`' will install the package's files in '`usr/local/bin`', '`usr/local/lib`', etc. You can specify an installation prefix other than '`/usr/local`' by giving '`configure`' the option '`--prefix=PATH`'. Alternately, you can do so by consistently giving a value for the '`prefix`' variable when you run '`make`', e.g.

```
make prefix=/usr/gnu
make prefix=/usr/gnu install
```

After installing you can remove the program binaries and object files from the source directory by typing '`make clean`'. To remove all files that '`configure`' created ('`Makefile`', '`config.status`', etc.), just type '`make distclean`'.

The file '`configure.ac`' is used to create '`configure`' by a program called '`autoconf`'. You only need it if you want to remake '`configure`' using a newer version of '`autoconf`'.

A.7 Uninstalling the package

To uninstall the GLPK package, i.e. to remove all the package's files from the system places, you can use the following command:

```
make uninstall
```

Appendix B

MPS Format

B.1 Fixed MPS Format

The MPS format[1] is intended for coding LP/MIP problem data. This format assumes the formulation of LP/MIP problem (1.1)—(1.3) (see Section 1.1, page 13).

MPS file is a text file, which contains two types of cards[2]: indicator cards and data cards.

Indicator cards determine a kind of succeeding data. Each indicator card has one word in uppercase letters beginning in column 1.

Data cards contain problem data. Each data card is divided into six fixed fields:

	Field 1	Field 2	Field 3	Field 4	Field 5	Feld 6
Columns	2—3	5—12	15—22	25—36	40—47	50—61
Contents	Code	Name	Name	Number	Name	Number

On a particular data card some fields may be optional.

Names are used to identify rows, columns, and some vectors (see below).

Aligning the indicator code in the field 1 to the left margin is optional.

All names specified in the fields 2, 3, and 5 should contain from 1 up to 8 arbitrary characters (except control characters). If a name is placed in the

[1] The MPS format was developed in 1960's by IBM as input format for their mathematical programming system MPS/360. Today the MPS format is a most widely used format understood by most mathematical programming packages. This appendix describes only the features of the MPS format, which are implemented in the GLPK package.

[2] In 1960's MPS file was a deck of 80-column punched cards, so the author decided to keep the word "card", which may be understood as "line of text file".

field 3 or 5, its first character should not be the dollar sign '$'. If a name contains spaces, the spaces are ignored.

All numerical values in the fields 4 and 6 should be coded in the form sxxEsyy, where s is the plus '+' or the minus '-' sign, xx is a real number with optional decimal point, yy is an integer decimal exponent. Any number should contain up to 12 characters. If the sign s is omitted, the plus sign is assumed. The exponent part is optional. If a number contains spaces, the spaces are ignored.

If a card has the asterisk '*' in the column 1, this card is considered as a comment and ignored. Besides, if the first character in the field 3 or 5 is the dollar sign '$', all characters from the dollar sign to the end of card are considered as a comment and ignored.

MPS file should contain cards in the following order:

- NAME indicator card;
- ROWS indicator card;
- data cards specifying rows (constraints);
- COLUMNS indicator card;
- data cards specifying columns (structural variables) and constraint coefficients;
- RHS indicator card;
- data cards specifying right-hand sides of constraints;
- RANGES indicator card;
- data cards specifying ranges for double-bounded constraints;
- BOUNDS indicator card;
- data cards specifying types and bounds of structural variables;
- ENDATA indicator card.

Section is a group of cards consisting of an indicator card and data cards succeeding this indicator card. For example, the ROWS section consists of the ROWS indicator card and data cards specifying rows.

The sections RHS, RANGES, and BOUNDS are optional and may be omitted.

B.2 Free MPS Format

Free MPS format is an improved version of the standard (fixed) MPS format described above.[3] Note that all changes in free MPS format concern only

[3] This format was developed in the beginning of 1990's by IBM as an alternative to the standard fixed MPS format for Optimization Subroutine Library (OSL).

the coding of data while the structure of data is the same for both fixed and free versions of the MPS format.

In free MPS format indicator and data records[4] may have arbitrary length not limited to 80 characters. Fields of data records have no predefined positions, i.e. the fields may begin in any position, except position 1, which must be blank, and must be separated from each other by one or more blanks. However, the fields must appear in the same order as in fixed MPS format.

Symbolic names in fields 2, 3, and 5 may be longer than 8 characters[5] and must not contain embedded blanks.

Numeric values in fields 4 and 6 are limited to 12 characters and must not contain embedded blanks.

Only six fields on each data record are used. Any other fields are ignored.

If the first character of any field (not necessarily fields 3 and 5) is the dollar sign ($), all characters from the dollar sign to the end of record are considered as a comment and ignored.

B.3 NAME indicator card

The NAME indicator card should be the first card in the MPS file (except optional comment cards, which may precede the NAME card). This card should contain the word `NAME` in the columns 1—4 and the problem name in the field 3. The problem name is optional and may be omitted.

B.4 ROWS section

The ROWS section should start with the indicator card, which contains the word `ROWS` in the columns 1—4.

Each data card in the ROWS section specifies one row (constraint) of the problem. All these data cards have the following format.

'`N`' in the field 1 means that the row is free (unbounded):

$$-\infty < x_i = a_{i1}x_{m+1} + a_{i2}x_{m+2} + \ldots + a_{in}x_{m+n} < +\infty;$$

'`L`' in the field 1 means that the row is of "less than or equal to" type:

$$-\infty < x_i = a_{i1}x_{m+1} + a_{i2}x_{m+2} + \ldots + a_{in}x_{m+n} \leq b_i;$$

[4] *Record* in free MPS format has the same meaning as *card* in fixed MPS format.

[5] GLPK allows symbolic names having up to 255 characters.

'`G`' in the field 1 means that the row is of "greater than or equal to" type:

$$b_i \leq x_i = a_{i1}x_{m+1} + a_{i2}x_{m+2} + \ldots + a_{in}x_{m+n} < +\infty;$$

'`E`' in the field 1 means that the row is of "equal to" type:

$$x_i = a_{i1}x_{m+1} + a_{i2}x_{m+2} + \ldots + a_{in}x_{m+n} \leq b_i,$$

where b_i is a right-hand side. Note that each constraint has a corresponding implictly defined auxiliary variable (x_i above), whose value is a value of the corresponding linear form, therefore row bounds can be considered as bounds of such auxiliary variable.

The filed 2 specifies a row name (which is considered as the name of the corresponding auxiliary variable).

The fields 3, 4, 5, and 6 are not used and should be empty.

Numerical values of all non-zero right-hand sides b_i should be specified in the RHS section (see below). All double-bounded (ranged) constraints should be specified in the RANGES section (see below).

B.5 COLUMNS section

The COLUMNS section should start with the indicator card, which contains the word `COLUMNS` in the columns 1—7.

Each data card in the COLUMNS section specifies one or two constraint coefficients a_{ij} and also introduces names of columns, i.e. names of structural variables. All these data cards have the following format.

The field 1 is not used and should be empty.

The field 2 specifies a column name. If this field is empty, the column name from the immediately preceeding data card is assumed.

The field 3 specifies a row name defined in the ROWS section.

The field 4 specifies a numerical value of the constraint coefficient a_{ij}, which is placed in the corresponding row and column.

The fields 5 and 6 are optional. If they are used, they should contain a second pair "row name—constraint coefficient" for the same column.

Elements of the constraint matrix (i.e. constraint coefficients) should be enumerated in the column wise manner: all elements for the current column should be specified before elements for the next column. However, the order of rows in the COLUMNS section may differ from the order of rows in the ROWS section.

Constraint coefficients not specified in the COLUMNS section are considered as zeros. Therefore zero coefficients may be omitted, although it is allowed to explicitly specify them.

B.6 RHS section

The RHS section should start with the indicator card, which contains the word `RHS` in the columns 1—3.

Each data card in the RHS section specifies one or two right-hand sides b_i (see Section B.4, page 238). All these data cards have the following format.

The field 1 is not used and should be empty.

The field 2 specifies a name of the right-hand side (RHS) vector[6]. If this field is empty, the RHS vector name from the immediately preceeding data card is assumed.

The field 3 specifies a row name defined in the ROWS section.

The field 4 specifies a right-hand side b_i for the row, whose name is specified in the field 3. Depending on the row type b_i is a lower bound (for the row of `G` type), an upper bound (for the row of `L` type), or a fixed value (for the row of `E` type).[7]

The fields 5 and 6 are optional. If they are used, they should contain a second pair "row name—right-hand side" for the same RHS vector.

All right-hand sides for the current RHS vector should be specified before right-hand sides for the next RHS vector. However, the order of rows in the RHS section may differ from the order of rows in the ROWS section.

Right-hand sides not specified in the RHS section are considered as zeros. Therefore zero right-hand sides may be omitted, although it is allowed to explicitly specify them.

B.7 RANGES section

The RANGES section should start with the indicator card, which contains the word `RANGES` in the columns 1—6.

Each data card in the RANGES section specifies one or two ranges for double-side constraints, i.e. for constraints that are of the types `L` and `G` at the same time:

$$l_i \le x_i = a_{i1}x_{m+1} + a_{i2}x_{m+2} + \ldots + a_{in}x_{m+n} \le u_i,$$

where l_i is a lower bound, u_i is an upper bound. All these data cards have the following format.

[6] This feature allows the user to specify several RHS vectors in the same MPS file. However, before solving the problem a particular RHS vector should be chosen.

[7] If the row is of `N` type, b_i is considered as a constant term of the corresponding linear form. Should note, however, this convention is non-standard.

The field 1 is not used and should be empty.

The field 2 specifies a name of the range vector[8]. If this field is empty, the range vector name from the immediately preceeding data card is assumed.

The field 3 specifies a row name defined in the ROWS section.

The field 4 specifies a range value r_i (see the table below) for the row, whose name is specified in the field 3.

The fields 5 and 6 are optional. If they are used, they should contain a second pair "row name—range value" for the same range vector.

All range values for the current range vector should be specified before range values for the next range vector. However, the order of rows in the RANGES section may differ from the order of rows in the ROWS section.

For each double-side constraint specified in the RANGES section its lower and upper bounds are determined as follows:

Row type	Sign of r_i	Lower bound	Upper bound
`G`	$+$ or $-$	b_i	$b_i + \|r_i\|$
`L`	$+$ or $-$	$b_i - \|r_i\|$	b_i
`E`	$+$	b_i	$b_i + \|r_i\|$
`E`	$-$	$b_i - \|r_i\|$	b_i

where b_i is a right-hand side specified in the RHS section (if b_i is not specified, it is considered as zero), r_i is a range value specified in the RANGES section.

B.8 BOUNDS section

The BOUNDS section should start with the indicator card, which contains the word `BOUNDS` in the columns 1—6.

Each data card in the BOUNDS section specifies one (lower or upper) bound for one structural variable (column). All these data cards have the following format.

The indicator in the field 1 specifies the bound type:

`LO` lower bound;

`UP` upper bound;

`FX` fixed variable (lower and upper bounds are equal);

`FR` free variable (no bounds);

`MI` no lower bound (lower bound is "minus infinity");

`PL` no upper bound (upper bound is "plus infinity");

[8]This feature allows the user to specify several range vectors in the same MPS file. However, before solving the problem a particular range vector should be chosen.

The field 2 specifies a name of the bound vector[9]. If this field is empty, the bound vector name from the immediately preceeding data card is assumed.

The field 3 specifies a column name defined in the COLUMNS section.

The field 4 specifies a bound value. If the bound type in the field 1 differs from `LO`, `UP`, and `FX`, the value in the field 4 is ignored and may be omitted.

The fields 5 and 6 are not used and should be empty.

All bound values for the current bound vector should be specified before bound values for the next bound vector. However, the order of columns in the BOUNDS section may differ from the order of columns in the COLUMNS section. Specification of a lower bound should precede specification of an upper bound for the same column (if both the lower and upper bounds are explicitly specified).

By default, all columns (structural variables) are non-negative, i.e. have zero lower bound and no upper bound. Lower (l_j) and upper (u_j) bounds of some column (structural variable x_j) are set in the following way, where s_j is a corresponding bound value explicitly specified in the BOUNDS section:

`LO` sets l_j to s_j;

`UP` sets u_j to s_j;

`FX` sets both l_j and u_j to s_j;

`FR` sets l_j to $-\infty$ and u_j to $+\infty$;

`MI` sets l_j to $-\infty$;

`PL` sets u_j to $+\infty$.

B.9 ENDATA indicator card

The ENDATA indicator card should be the last card of MPS file (except optional comment cards, which may follow the ENDATA card). This card should contain the word `ENDATA` in the columns 1—6.

B.10 Specifying objective function

It is impossible to explicitly specify the objective function and optimization direction in the MPS file. However, the following implicit rule is used by default: the first row of `N` type is considered as a row of the objective function (i.e. the objective function is the corresponding auxiliary variable), which should be *minimized*.

[9] This feature allows the user to specify several bound vectors in the same MPS file. However, before solving the problem a particular bound vector should be chosen.

GLPK also allows specifying a constant term of the objective function as a right-hand side of the corresponding row in the RHS section.

B.11 Example of MPS file

In order to illustrate what the MPS format is, consider the following example of LP problem:

minimize

$$value = .03\ bin_1 + .08\ bin_2 + .17\ bin_3 + .12\ bin_4 + .15\ bin_5 + .21\ al + .38\ si$$

subject to linear constraints

$$\begin{aligned}
yield &= bin_1 + bin_2 + bin_3 + bin_4 + bin_5 + al + si \\
FE &= .15\ bin_1 + .04\ bin_2 + .02\ bin_3 + .04\ bin_4 + .02\ bin_5 + .01\ al + .03\ si \\
CU &= .03\ bin_1 + .05\ bin_2 + .08\ bin_3 + .02\ bin_4 + .06\ bin_5 + .01\ al \\
MN &= .02\ bin_1 + .04\ bin_2 + .01\ bin_3 + .02\ bin_4 + .02\ bin_5 \\
MG &= .02\ bin_1 + .03\ bin_2 + .01\ bin_5 \\
AL &= .70\ bin_1 + .75\ bin_2 + .80\ bin_3 + .75\ bin_4 + .80\ bin_5 + .97\ al \\
SI &= .02\ bin_1 + .06\ bin_2 + .08\ bin_3 + .12\ bin_4 + .02\ bin_5 + .01\ al + .97\ si
\end{aligned}$$

and bounds of (auxiliary and structural) variables

$$\begin{aligned}
yield &= 2000 & 0 &\le bin_1 \le 200 \\
-\infty < FE &\le 60 & 0 &\le bin_2 \le 2500 \\
-\infty < CU &\le 100 & 400 &\le bin_3 \le 800 \\
-\infty < MN &\le 40 & 100 &\le bin_4 \le 700 \\
-\infty < MG &\le 30 & 0 &\le bin_5 \le 1500 \\
1500 \le AL &< +\infty & 0 &\le al < +\infty \\
250 \le SI &\le 300 & 0 &\le si < +\infty
\end{aligned}$$

A complete MPS file which specifies data for this example is shown below (the first two comment lines show card positions).

```
*00000000111111111122222222223333333333444444444455555555566
*23456789012345678901234567890123456789012345678901234567890
NAME          PLAN
ROWS
 N  VALUE
 E  YIELD
 L  FE
```

```
 L  CU
 L  MN
 L  MG
 G  AL
 L  SI
COLUMNS
    BIN1      VALUE          .03000   YIELD         1.00000
              FE             .15000   CU             .03000
              MN             .02000   MG             .02000
              AL             .70000   SI             .02000
    BIN2      VALUE          .08000   YIELD         1.00000
              FE             .04000   CU             .05000
              MN             .04000   MG             .03000
              AL             .75000   SI             .06000
    BIN3      VALUE          .17000   YIELD         1.00000
              FE             .02000   CU             .08000
              MN             .01000   AL             .80000
              SI             .08000
    BIN4      VALUE          .12000   YIELD         1.00000
              FE             .04000   CU             .02000
              MN             .02000   AL             .75000
              SI             .12000
    BIN5      VALUE          .15000   YIELD         1.00000
              FE             .02000   CU             .06000
              MN             .02000   MG             .01000
              AL             .80000   SI             .02000
    ALUM      VALUE          .21000   YIELD         1.00000
              FE             .01000   CU             .01000
              AL             .97000   SI             .01000
    SILICON   VALUE          .38000   YIELD         1.00000
              FE             .03000   SI             .97000
RHS
    RHS1      YIELD       2000.00000   FE           60.00000
              CU           100.00000   MN           40.00000
              SI           300.00000
              MG            30.00000   AL         1500.00000
RANGES
    RNG1      SI            50.00000
BOUNDS
 UP BND1      BIN1         200.00000
```

```
 UP           BIN2        2500.00000
 LO           BIN3         400.00000
 UP           BIN3         800.00000
 LO           BIN4         100.00000
 UP           BIN4         700.00000
 UP           BIN5        1500.00000
ENDATA
```

B.12 MIP features

The MPS format provides two ways for introducing integer variables into the problem.

The first way is most general and based on using special marker cards INTORG and INTEND. These marker cards are placed in the COLUMNS section. The INTORG card indicates the start of a group of integer variables (columns), and the card INTEND indicates the end of the group. The MPS file may contain arbitrary number of the marker cards.

The marker cards have the same format as the data cards (see Section B.1, page 236).

The fields 1, 2, and 6 are not used and should be empty.

The field 2 should contain a marker name. This name may be arbitrary.

The field 3 should contain the word `'MARKER'` (including apostrophes).

The field 5 should contain either the word `'INTORG'` (including apostrophes) for the marker card, which begins a group of integer columns, or the word `'INTEND'` (including apostrophes) for the marker card, which ends the group.

The second way is less general but more convenient in some cases. It allows the user declaring integer columns using three additional types of bounds, which are specified in the field 1 of data cards in the BOUNDS section (see Section B.8, page 241):

`LI` lower integer. This bound type specifies that the corresponding column (structural variable), whose name is specified in field 3, is of integer kind. In this case an lower bound of the column should be specified in field 4 (like in the case of `LO` bound type).

`UI` upper integer. This bound type specifies that the corresponding column (structural variable), whose name is specified in field 3, is of integer kind. In this case an upper bound of the column should be specified in field 4 (like in the case of `UP` bound type).

`BV` binary variable. This bound type specifies that the corresponding column (structural variable), whose name is specified in the field 3, is of integer kind, its lower bound is zero, and its upper bound is one (thus, such variable being of integer kind can have only two values zero and one). In this case a numeric value specified in the field 4 is ignored and may be omitted.

Consider the following example of MIP problem:

minimize

$$Z = 3x_1 + 7x_2 - x_3 + x4$$

subject to linear constraints

$$\begin{aligned} r_1 &= 2x_1 - x_2 + x_3 - x_4 \\ r_2 &= x_1 - x_2 - 6x_3 + 4x_4 \\ r_3 &= 5x_1 + 3x_2 + x_4 \end{aligned}$$

and bound of variables

$$\begin{array}{lll} 1 \le r_1 < +\infty & 0 \le x_1 \le 4 & \text{(continuous)} \\ 8 \le r_2 < +\infty & 2 \le x_2 \le 5 & \text{(integer)} \\ 5 \le r_3 < +\infty & 0 \le x_3 \le 1 & \text{(integer)} \\ & 3 \le x_4 \le 8 & \text{(continuous)} \end{array}$$

The corresponding MPS file may look like the following:

```
NAME          SAMP1
ROWS
 N  Z
 G  R1
 G  R2
 G  R3
COLUMNS
    X1        R1                 2.0    R2                 1.0
    X1        R3                 5.0    Z                  3.0
    MARK0001  'MARKER'                  'INTORG'
    X2        R1                -1.0    R2                -1.0
    X2        R3                 3.0    Z                  7.0
    X3        R1                 1.0    R2                -6.0
    X3        Z                 -1.0
    MARK0002  'MARKER'                  'INTEND'
    X4        R1                -1.0    R2                 4.0
```

```
    X4        R3                1.0    Z                 1.0
RHS
    RHS1      R1                1.0
    RHS1      R2                8.0
    RHS1      R3                5.0
BOUNDS
 UP BND1      X1                4.0
 LO BND1      X2                2.0
 UP BND1      X2                5.0
 UP BND1      X3                1.0
 LO BND1      X4                3.0
 UP BND1      X4                8.0
ENDATA
```

The same example may be coded without INTORG/INTEND markers using the bound type UI for the variable x_2 and the bound type BV for the variable x_3:

```
NAME          SAMP2
ROWS
 N  Z
 G  R1
 G  R2
 G  R3
COLUMNS
    X1        R1                2.0    R2                1.0
    X1        R3                5.0    Z                 3.0
    X2        R1               -1.0    R2               -1.0
    X2        R3                3.0    Z                 7.0
    X3        R1                1.0    R2               -6.0
    X3        Z                -1.0
    X4        R1               -1.0    R2                4.0
    X4        R3                1.0    Z                 1.0
RHS
    RHS1      R1                1.0
    RHS1      R2                8.0
    RHS1      R3                5.0
BOUNDS
 UP BND1      X1                4.0
 LO BND1      X2                2.0
```

```
 UI BND1      X2                5.0
 BV BND1      X3
 LO BND1      X4                3.0
 UP BND1      X4                8.0
ENDATA
```

Appendix C

CPLEX LP Format

C.1 Prelude

The CPLEX LP format[1] is intended for coding LP/MIP problem data. It is a row-oriented format that assumes the formulation of LP/MIP problem (1.1)—(1.3) (see Section 1.1, page 13).

CPLEX LP file is a plain text file written in CPLEX LP format. Each text line of this file may contain up to 255 characters[2]. Blank lines are ignored. If a line contains the backslash character (\), this character and everything that follows it until the end of line are considered as a comment and also ignored.

An LP file is coded by the user using the following elements:

- keywords;
- symbolic names;
- numeric constants;
- delimiters;
- blanks.

[1] The CPLEX LP format was developed in the end of 1980's by CPLEX Optimization, Inc. as an input format for the CPLEX linear programming system. Although the CPLEX LP format is not as widely used as the MPS format, being row-oriented it is more convenient for coding mathematical programming models by human. This appendix describes only the features of the CPLEX LP format which are implemented in the GLPK package.

[2] GLPK allows text lines of arbitrary length.

Keywords which may be used in the LP file are the following:

```
minimize        minimum         min
maximize        maximum         max
subject to      such that       s.t.      st.      st
bounds          bound
general         generals        gen
integer         integers        int
binary          binaries        bin
infinity        inf
free
end
```

All the keywords are case insensitive. Keywords given above on the same line are equivalent. Any keyword (except `infinity`, `inf`, and `free`) being used in the LP file must start at the beginning of a text line.

Symbolic names are used to identify the objective function, constraints (rows), and variables (columns). All symbolic names are case sensitive and may contain up to 16 alphanumeric characters[3] (`a`, ..., `z`, `A`, ..., `Z`, `0`, ..., `9`) as well as the following characters:

```
!  "  #  $  %  &  (  )  /  ,  .  ;  ?  @  _  `  '  {  }  |  ~
```

with exception that no symbolic name can begin with a digit or a period.

Numeric constants are used to denote constraint and objective coefficients, right-hand sides of constraints, and bounds of variables. They are coded in the standard form xx`E`$s yy$, where xx is a real number with optional decimal point, s is a sign (`+` or `-`), yy is an integer decimal exponent. Numeric constants may contain arbitrary number of characters. The exponent part is optional. The letter '`E`' can be coded as '`e`'. If the sign s is omitted, plus is assumed.

Delimiters that may be used in the LP file are the following:

```
:
+
-
<   <=   =<
>   >=   =>
=
```

[3] GLPK allows symbolic names having up to 255 characters.

Delimiters given above on the same line are equivalent. The meaning of the delimiters will be explained below.

Blanks are non-significant characters. They may be used freely to improve readability of the LP file. Besides, blanks should be used to separate elements from each other if there is no other way to do that (for example, to separate a keyword from a following symbolic name).

The order of an LP file is:

- objective function definition;
- constraints section;
- bounds section;
- general, integer, and binary sections (can appear in arbitrary order);
- end keyword.

These components are discussed in following sections.

C.2 Objective function definition

The objective function definition must appear first in the LP file. It defines the objective function and specifies the optimization direction.

The objective function definition has the following form:

$$\left\{\begin{matrix}\texttt{minimize}\\ \texttt{maximize}\end{matrix}\right\}\ f : s\ c\ x\ s\ c\ x\ \ldots\ s\ c\ x$$

where f is a symbolic name of the objective function, s is a sign `+` or `-`, c is a numeric constant that denotes an objective coefficient, x is a symbolic name of a variable.

If necessary, the objective function definition can be continued on as many text lines as desired.

The name of the objective function is optional and may be omitted (together with the semicolon that follows it). In this case the default name '`obj`' is assigned to the objective function.

If the very first sign s is omitted, the sign plus is assumed. Other signs cannot be omitted.

If some objective coefficient c is omitted, 1 is assumed.

Symbolic names x used to denote variables are recognized by context and therefore needn't to be declared somewhere else.

Here is an example of the objective function definition:

```
Minimize Z : - x1 + 2 x2 - 3.5 x3 + 4.997e3x(4) + x5 + x6 +
   x7 - .01x8
```

C.3 Constraints section

The constraints section must follow the objective function definition. It defines a system of equality and/or inequality constraints.

The constraint section has the following form:

```
subject to
constraint_1
constraint_2
   ...
constraint_m
```

where $constraint_i, i = 1, \ldots, m$, is a particular constraint definition.

Each constraint definition can be continued on as many text lines as desired. However, each constraint definition must begin on a new line except the very first constraint definition which can begin on the same line as the keyword '`subject to`'.

Constraint definitions have the following form:

$$r : s\ c\ x\ s\ c\ x\ \ldots\ s\ c\ x\ \left\{\begin{array}{l}\texttt{<=}\\ \texttt{>=}\\ \texttt{=}\end{array}\right\}\ b$$

where r is a symbolic name of a constraint, s is a sign `+` or `-`, c is a numeric constant that denotes a constraint coefficient, x is a symbolic name of a variable, b is a right-hand side.

The name r of a constraint (which is the name of the corresponding auxiliary variable) is optional and may be omitted (together with the semicolon that follows it). In this case the default names like '`r.nnn`' are assigned to unnamed constraints.

The linear form $s\ c\ x\ s\ c\ x\ \ldots\ s\ c\ x$ in the left-hand side of a constraint definition has exactly the same meaning as in the case of the objective function definition (see above).

After the linear form one of the following delimiters that indicate the constraint sense must be specified:

`<=` means 'less than or equal to'

`>=` means 'greater than or equal to'

`=` means 'equal to'

The right hand side b is a numeric constant with an optional sign.

Here is an example of the constraints section:

```
Subject To
   one: y1 + 3 a1 - a2 - b >= 1.5
   y2 + 2 a3 + 2
      a4 - b >= -1.5
   two : y4 + 3 a1 + 4 a5 - b <= +1
   .20y5 + 5 a2 - b = 0
   1.7 y6 - a6 + 5 a777 - b >= 1
```

(Should note that it is impossible to express ranged constraints in the CPLEX LP format. Each a ranged constraint can be coded as two constraints with identical linear forms in the left-hand side, one of which specifies a lower bound and other does an upper one of the original ranged constraint.)

C.4 Bounds section

The bounds section is intended to define bounds of variables. This section is optional; if it is specified, it must follow the constraints section. If the bound section is omitted, all variables are assumed to be non-negative (i.e. that they have zero lower bound and no upper bound).

The bounds section has the following form:

`bounds`
definition$_1$
definition$_2$
...
definition$_p$

where *definition*$_k, k = 1, \ldots, p$, is a particular bound definition.

Each bound definition must begin on a new line[4] except the very first bound definition which can begin on the same line as the keyword '`bounds`'.

Syntactically constraint definitions can have one of the following six forms:

x `>=` l	specifies a lower bound
l `<=` x	specifies a lower bound
x `<=` u	specifies an upper bound
l `<=` x `<=` u	specifies both lower and upper bounds
x `=` t	specifies a fixed value
x `free`	specifies free variable

[4]The GLPK implementation allows several bound definitions to be placed on the same line.

where x is a symbolic name of a variable, l is a numeric constant with an optional sign that defines a lower bound of the variable or `-inf` that means that the variable has no lower bound, u is a numeric constant with an optional sign that defines an upper bound of the variable or `+inf` that means that the variable has no upper bound, t is a numeric constant with an optional sign that defines a fixed value of the variable.

By default all variables are non-negative, i.e. have zero lower bound and no upper bound. Therefore definitions of these default bounds can be omitted in the bounds section.

Here is an example of the bounds section:

```
Bounds
   -inf <= a1 <= 100
   -100 <= a2
   b <= 100
   x2 = +123.456
   x3 free
```

C.5 General, integer, and binary sections

The general, integer, and binary sections are intended to define some variables as integer or binary. All these sections are optional and needed only in case of MIP problems. If they are specified, they must follow the bounds section or, if the latter is omitted, the constraints section.

All the general, integer, and binary sections have the same form as follows:

$$\left\{\begin{matrix}\texttt{general}\\ \texttt{integer}\\ \texttt{binary}\end{matrix}\right\}$$
$$x_1$$
$$x_2$$
$$\dots$$
$$x_q$$

where x_k is a symbolic name of variable, $k = 1, \dots, q$.

Each symbolic name must begin on a new line[5] except the very first symbolic name which can begin on the same line as the keyword '`general`', '`integer`', or '`binary`'.

[5] The GLPK implementation allows several symbolic names to be placed on the same line.

If a variable appears in the general or the integer section, it is assumed to be general integer variable. If a variable appears in the binary section, it is assumed to be binary variable, i.e. an integer variable whose lower bound is zero and upper bound is one. (Note that if bounds of a variable are specified in the bounds section and then the variable appears in the binary section, its previously specified bounds are ignored.)

Here is an example of the integer section:

```
Integer
   z12
   z22
   z35
```

C.6 End keyword

The keyword '`end`' is intended to end the LP file. It must begin on a separate line and no other elements (except comments and blank lines) must follow it. Although this keyword is optional, it is strongly recommended to include it in the LP file.

C.7 Example of CPLEX LP file

Here is a complete example of CPLEX LP file that corresponds to the example given in Section B.11, page 243.

```
\* plan.lp *\

Minimize
   value: .03 bin1 + .08 bin2 + .17 bin3 + .12 bin4 + .15 bin5 +
          .21 alum + .38 silicon

Subject To
   yield:     bin1 +     bin2 +     bin3 +     bin4 +     bin5 +
              alum +     silicon                                 =  2000

   fe:    .15 bin1 + .04 bin2 + .02 bin3 + .04 bin4 + .02 bin5 +
          .01 alum + .03 silicon                                 <=   60

   cu:    .03 bin1 + .05 bin2 + .08 bin3 + .02 bin4 + .06 bin5 +
          .01 alum                                               <=  100

   mn:    .02 bin1 + .04 bin2 + .01 bin3 + .02 bin4 + .02 bin5   <=   40
```

```
   mg:    .02 bin1 + .03 bin2                      + .01 bin5   <=   30

   al:    .70 bin1 + .75 bin2 + .80 bin3 + .75 bin4 + .80 bin5 +
          .97 alum                                              >= 1500

   si1:   .02 bin1 + .06 bin2 + .08 bin3 + .12 bin4 + .02 bin5 +
          .01 alum + .97 silicon                                >=  250

   si2:   .02 bin1 + .06 bin2 + .08 bin3 + .12 bin4 + .02 bin5 +
          .01 alum + .97 silicon                                <=  300

Bounds
          bin1 <=  200
          bin2 <= 2500
   400 <= bin3 <=  800
   100 <= bin4 <=  700
          bin5 <= 1500

End

\* eof *\
```

Appendix D

Stand-alone LP/MIP Solver

The GLPK package includes the program `glpsol`, which is a stand-alone LP/MIP solver. This program can be invoked from the command line of from the shell to read LP/MIP problem data in any format supported by GLPK, solve the problem, and write the problem solution obtained to an output text file.

Usage

`glpsol` [*options...*] [*filename*]

General options

`--mps` read LP/MIP problem in fixed MPS format

`--freemps` read LP/MIP problem in free MPS format (default)

`--cpxlp` read LP/MIP problem in CPLEX LP format

`--math` read LP/MIP model written in GNU MathProg modeling language

`-m` *filename*, `--model` *filename*
read model section and optional data section from *filename* (the same as `--math`)

`-d` *filename*, `--data` *filename*
read data section from *filename* (for `--math` only); if model file also has data section, that section is ignored

`-y` *filename*, `--display` *filename*
send display output to *filename* (for `--math` only); by default the output is sent to `stdout`

`--mincost` read min-cost flow problem in DIMACS format
`--maxflow` read maximum flow problem in DIMACS format
`--simplex` use simplex method (default)
`--interior` use interior point method (for pure LP only)
`-r` *filename*, `--read` *filename*
read solution from *filename* rather to find it with the solver
`--min` minimization
`--max` maximization
`--scale` scale problem (default)
`--noscale` do not scale problem
`-o` *filename*, `--output` *filename*
write solution to *filename* in printable format
`-w` *filename*, `--write` *filename*
write solution to *filename* in plain text format
`--bounds` *filename*
write sensitivity bounds to *filename* in printable format (LP only)
`--tmlim` *nnn* limit solution time to *nnn* seconds (`--tmlim 0` allows obtaining solution at initial point)
`--memlim` *nnn* limit available memory to *nnn* Megabytes
`--check` do not solve problem, check input data only
`--name` *probname* change problem name to *probname*
`--wmps` *filename* write problem to *filename* in fixed MPS format
`--wfreemps` *filename*
write problem to *filename* in free MPS format
`--wcpxlp` *filename*
write problem to *filename* in CPLEX LP format
`--log` *filename* write copy of terminal output to *filename*
`-h`, `--help` display this help information and exit
`-v`, `--version` display program version and exit

LP basis factorization options

`--luf` LU + Forrest–Tomlin update
(faster, less stable; default)
`--cbg` LU + Schur complement + Bartels–Golub update
(slower, more stable)
`--cbg` LU + Schur complement + Givens rotation update
(slower, more stable)

Options specific to the simplex solver

`--primal`	use primal simplex (default)
`--dual`	use dual simplex
`--std`	use standard initial basis of all slacks
`--adv`	use advanced initial basis (default)
`--bib`	use Bixby's initial basis
`--steep`	use steepest edge technique (default)
`--nosteep`	use standard "textbook" pricing
`--relax`	use Harris' two-pass ratio test (default)
`--norelax`	use standard "textbook" ratio test
`--presol`	use LP presolver (default; assumes `--scale` and `--adv`)
`--nopresol`	do not use LP presolver
`--exact`	use simplex method based on exact arithmetic
`--xcheck`	check final basis using exact arithmetic

Options specific to the interior-point solver

`--nord`	use natural (original) ordering
`--qmd`	use quotient minimum degree ordering
`--amd`	use approximate minimum degree ordering (default)
`--symamd`	use approximate minimum degree ordering

Options specific to the MIP solver

`--nomip`	consider all integer variables as continuous (allows solving MIP as pure LP)
`--first`	branch on first integer variable
`--last`	branch on last integer variable
`--mostf`	branch on most fractional variable
`--drtom`	branch using heuristic by Driebeck and Tomlin (default)
`--pcost`	branch using hybrid pseudocost heuristic (may be useful for hard instances)
`--dfs`	backtrack using depth first search
`--bfs`	backtrack using breadth first search
`--bestp`	backtrack using the best projection heuristic (default)
`--bestb`	backtrack using node with best local bound
`--intopt`	use advanced MIP solver (enables MIP presolving)
`--binarize`	replace general integer variables by binary ones (assumes `--intopt`)
`--cover`	generate mixed cover cuts

`--clique`	generate clique cuts
`--gomory`	generate Gomory's mixed integer cuts
`--mir`	generate MIR (mixed integer rounding) cuts
`--cuts`	generate cuts of all classes above (assumes `--intopt`)

For description of the MPS format see Appendix B, page 236.

For description of the CPLEX LP format see Appendix C, page 249.

For description of the modeling language see the document "Modeling Language GNU MathProg: Language Reference" included in the GLPK distribution.

For description of the DIMACS min-cost flow problem format see subsection 6.6.2, page 169.

For description of the DIMACS maximum flow problem format see subsection 6.7.2, page 185.

Appendix E

External Software Modules Used In GLPK

In the GLPK package there are used some external software modules listed in this Appendix. Note that these modules are *not* part of GLPK, but are used with GLPK and included in the distribution.

E.1 AMD

Description

AMD is a set of routines for pre-ordering sparse matrices prior to Cholesky or LU factorization, using the approximate minimum degree ordering algorithm.

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Availability

`http://www.cise.ufl.edu/research/sparse/amd`

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E.2 COLAMD/SYMAMD

Description

colamd: an approximate minimum degree column ordering algorithm, for LU factorization of symmetric or unsymmetric matrices, QR factorization, least squares, interior point methods for linear programming problems, and other related problems.

symamd: an approximate minimum degree ordering algorithm for Cholesky factorization of symmetric matrices.

Authors

The authors of the code itself are Stefan I. Larimore and Timothy A. Davis (davis at cise.ufl.edu), University of Florida. The algorithm was developed in collaboration with John Gilbert, Xerox PARC, and Esmond Ng, Oak Ridge National Laboratory.

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Version 3, 29 June 2007

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